# The segregative properties of endogenous jurisdictions formation with a welfarist central government*

Rongili Biswas[†], Nicolas Gravel[‡] and Rémy Oddou[§]

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**Abstract**

This paper examines the segregative properties of endogenous processes of jurisdiction formation *à la* Tiebout in the presence of a central government which makes equalization transfers to jurisdictions in such a way as to maximize a welfarist objective. Choice of location by households, of local public good provision by jurisdictions, and of equalization grants and tax by the central government are assumed to be made simultaneously, taking the choices of others as given. Two welfarist objectives for the central government are considered in turn: maxmin and generalized utilitarianism. If the central government pursues a maxmin objective, it is easily shown that the only stable jurisdiction structures that can emerge are those in which the jurisdictions' poorest households have all the same wealth. A richer class of stable jurisdiction structures are compatible with a central government which uses a generalized utilitarian objective. Yet, it turns out that, if the analysis is restricted to households with additively separable preferences, the class of such preferences that garantee the wealth segregation of any stable jurisdiction structure is unaffected by the presence of a central government.

## 1 Introduction

There is a wide presumption that decentralized processes of jurisdiction formation *à la* Tiebout (1956) lead individuals to self-sort into homogenous

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†POLIS, University of Eastern Piedmont, Via Cavour 84, 15 000 Alessandria, Italy

‡University of the Mediterranean and IDEP-GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, 13 002 Marseille, France.

§University of the Mediterranean and IDEP-GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, 13 002 Marseille, France.

communities. In a recent paper, Gravel and Thoron (2007) investigate the validity of this intuition within the classical model of jurisdictions formation developed by Westhoff (1977) (see also Greenberg and Weber (1986) and Demange (1994) among many others). In this model, unequally wealthy households with the same preference for a local public good and a private good choose simultaneously their place of residence in a finite set of locations. Households who choose the same location form a *jurisdiction* and produce a local public good by applying a democratically chosen tax rate to all residents' wealth. Any such simultaneous choice of residence by households is referred to as a *jurisdiction structure*.

The analysis of Gravel and Thoron (2007) concerns *stable* jurisdiction structures, which satisfy the additional property of being *robust to individual deviations*. The question raised by Gravel and Thoron (2007) is whether stable jurisdiction structures lead households to self sort, or segregate, themselves according to their wealth. The notion of segregation used is that known under the heading of *consecutiveness* in the coalition formation literature (see e.g. Greenberg and Weber (1986)). A jurisdiction structure is segregated in this sense if, for any two jurisdictions with different *per capita* wealth, the richest individual in the poorer jurisdiction is (weakly) poorer than the poorest individual in the richer jurisdiction. Gravel and Thoron (2007) identify a condition on households preferences that is necessary and sufficient for the segregation of any stable jurisdiction structure. The condition requires that the public good is *always* a gross complement, or *always* a gross substitute for the private good. While stringent, and violated by several well-known preferences, including additively separable ones, this Gross Substitutability-Complementarity (GSC) condition is not implausible. For this reason, Gravel and Thoron (2007) seems to provide theoretical ground for the belief that decentralized processes of jurisdiction formation are inherently segregative.

In this paper, we investigate the extent to which this conclusion is affected by the introduction of a *central government*. Introducing a central government in models of endogenous jurisdiction formation strikes us as an important step toward improving the realism of these models. In many countries, one finds indeed a juxtaposition of several levels of governments: central and local. It is also commonly observed that the central government puts into place *equalization payment schemes* which aims at redistributing funds across jurisdictions so as to achieve specific normative objectives. It is therefore interesting to examine the consequence of central government's intervention on the segregative properties of the endogenous formation of local jurisdictions by freely mobile households.

Doing this requires one to specify:

1) the instruments available to the central government

2) the objective of the central government and

3) the nature of the interaction between central government, households

and local governments.

As for the first point, we assume that the central government tax households at a fixed rate and redistributes tax revenues between jurisdictions in such a way as to maximize some objective function. Although stylized, this modeling of the redistribution performed by the central government does not provide a bad approximation of many existing systems of equalization payments observed in practice. It is consistent both with the so-called *horizontal* equalization payments scheme of the sort existing in Scandinavian countries and Germany - and the *vertical* schemes observed in several other countries (like Belgium, France, Canada, Australia, Switzerland and India, to mention just a few).[1]

As for the objective of the central government, we assume it to be welfarist. We consider more specifically two families of welfarist objectives. The first is the *generalized utilitarian* one which compares alternative packages of equalization grants and tax rates on the basis of the sum of an increasing and concave transformation of the households' utilities. This family, characterized in classical choice theory by plausible axioms (see e.g. Blackorby, Bossert, and Donaldson (2005), ch. 4), contains the classical utilitarian criterion as well as several others like, for instance, the symmetric mean of order $r$). The other welfarist criterion, that can be approached in the limit by the symmetric mean of order $r$ family, is the *maxmin* criterion that focuses on the utility of the worst off household.

As for the interaction between households, central and local governments, we model it as taking place *simultaneously*. We define accordingly a stable jurisdiction structure with a central government to be an assignment, to every location, of a local tax rate, a central government net transfer, and a set of households that is immune to individuals deviation from the part of all agents.

The question addressed in this paper is whether the GSC condition on households' preferences identified in Gravel and Thoron (2007) remains necessary and sufficient for ensuring the segregation of any stable jurisdiction structure with a central welfarist government. We examine this question by considering in turn a maxmin and a generalized utilitarian central government, making, in the later case, the extra assumption that households preferences, assumed to be represented by the utility function used by the central government, is additively separable. There is not much analysis to be performed in the case of a maxmin government. For it is shown easily in that case that the only stable jurisdiction structures are those in which all jurisdictions' poorest households have the same wealth and where, as a result, it is optimal for the central government to perform no equalization (and to raise

[1] A normative analysis of equalization payment in federations can be found in Gravel and Poitevin (2006) while a broader discussion of equalization in structures with multiple levels of governance is provided by Boadway (2006).

therefore no tax). Results are different with a generalized utilitarian central government because the optimal redistribution by such a government is compatible with a wider class of stable jurisdiction structures. Yet we show that, if households' preferences are additively separable, the GSC condition remains necessary and sufficient for the segregation of any stable jurisdiction structure with any generalized utilitarian central government. Hence, it appears that at least for households with additively separable preference, the presence of a generalized utilitarian central government does not affect the segregative properties of decentralized processes of jurisdiction formation in the Westhoff model.

The rest of the paper is organized as follows. The next section introduces the notation and the model and presents the result for the maxmin government. Section 3 states and proves the main result for the generalized utilitarian government and section 4 concludes.

# 2 The model

As in Gravel and Thoron (2007) and Westhoff (1977), we consider economies with a continuum of households represented by the $[0,1]$ interval. Any such economy consists of four elements.

*First*, there is a Lebesgue measure $\lambda$ on $[0,1]$. For any Lebesgue measurable subset $I$ of $[0,1]$, we interpret $\lambda(I)$ as "the mass of households" in the set $I$.

The *second* ingredient is a *wealth distribution* modeled as a Lebesgue measurable, increasing and bounded from above function $\omega : [0,1] \to \mathbb{R}_{++}$ that associates to each household $i \in [0,1]$ its strictly positive private wealth $\omega_i$. Assuming the function $\omega$ to be increasing is a convention according to which households are ordered by their wealth ($i \leq i' \Longrightarrow \omega_i \leq \omega_{i'}$).

The *third* ingredient is a specification of the household's preferences, taken to be the same for all households. We assume that household's preference for the local public good ($Z$) and the private good ($x$) is represented by a twice differentiable, strictly increasing and strictly concave utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$ bounded from below.[2] For the result concerning the generalized utilitarian central government, we make the additional assumption that the utility function that represents the household's preference is *additively separable* so that it can be written, for every $(\overline{Z}, \overline{x}) \in \mathbb{R}^2_+$, as:

$$U(\overline{Z}, \overline{x}) = f(\overline{Z}) + h(\overline{x}) \quad (1)$$

[2] The requirement of strict concavity is a simplifying assumption that guarantees the uniqueness of the solution to the standard consumer's program as well as the sufficiency of the first order conditions of the maximization program solved by a generalized utilitarian government. The assumption that the utility is bounded from below guarantees that the maxmin criterion used by the government is well-defined.

for some twice differentiable increasing and concave real valued functions $f$ and $h$ having both $\mathbb{R}_+$ as domain. Given any bundle of public and private good $(\overline{Z}, \overline{x}) \in \mathbb{R}^2_+$, we denote by $MRS(\overline{Z}, \overline{x})$ the *marginal rate of substitution of public good to private good* evaluated at $(\overline{Z}, \overline{x})$ defined by:

$$MRS(\overline{Z}, \overline{x}) = \frac{\partial U(\overline{Z}, \overline{x})/\partial Z}{\partial U(\overline{Z}, \overline{x})/\partial x} \tag{2}$$

We also denote by $Z^M(p_Z, p_x, R)$ and $x^M(p_Z, p_x, R)$ the households' Marshallian demands for the public and private good (respectively) when the prices of these goods are $p_Z$ and $p_x$ and the household's income is $R$. Marshallian demand functions are the (unique under our assumptions) solution of the program:

$$\max_{Z,x} U(Z, x) \text{ subject to } p_Z Z + p_x x \leq R$$

Given again our assumptions, Marshallian demands are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}^2_+$). We emphasize that we view Marshallian demands as dual representations of preferences rather than description of behavior (after all households rarely if ever purchase local public goods on competitive markets). The indirect utility function corresponding to $U$ is denoted by $V$ and is defined as usual by:

$$V(p_Z, p_x, R) = U(Z^M(p_Z, p_x, R), x^M(p_Z, p_x, R))$$

We further assume that Marshallian demand for public good satisfies the following additional *regularity* condition (introduced and discussed in Gravel and Thoron (2007)).

**Condition 1**: If there exists a public good price $\overline{p}_Z$, an income level $R$ and a non-degenerate interval $I$ of strictly positive real numbers such that, $Z^M(\overline{p}_Z, p_x, R) = Z^M(\overline{p}_Z, p'_x, R)$ for all prices $p'_x$ and $p_x$ in $I$, then, for all $(p_Z, p_x, R) \in \mathbb{R}^3_+$, we must have $Z^M(p_Z, p_x, R) = h(p_Z, R)$ for some function $h : \mathbb{R}^2_{++} \to \mathbb{R}_+$.
The *last* element in the description of an economy is a finite set $\mathbb{L} = \{1, ..., L\}$ of possible locations.

The problem considered is that of identifying the properties of the various *jurisdiction structures* that can emerge when households freely choose their location and share, when they locate at the same place, the benefit of a local public good produced by local tax revenues and a central government grant. This intervention of the central government is the main distinctive ingredient of our model as compared to what is done in the literature (e.g. Demange (1994), Gravel and Thoron (2007), Greenberg (1983), Westhoff (1977), Greenberg and Weber (1986), Wooders (1978) and Wooders (1999)).

We define a *jurisdiction structure* for the economy $(\lambda, \omega, U, \mathbb{L})$ as a Lebesgue measurable function $S : [0, 1] \to \mathbb{L}$, a (local) tax vector $t \in \mathbb{R}^L$, a central government grant vector $g \in \mathbb{R}^L$ and a central government tax rate $c \in [0,1]$ that satisfy :

1) $\sum_{l \in \mathbb{L}} g_l \leq c \int_{[0,1]} \omega_i d\lambda$ and, for every $l \in \mathbb{L}$,
2) $g_l \geq -\varpi_l$
3) $t_l \in [\frac{-g_l}{\varpi_l}, 1-c]$

where, for every location $l \in \mathbb{L}$, $\varpi_l = \int_{N_l^S} \omega_i d\lambda$ and $N_l^S = S^{-1}(l) = \{i \in [0,1] : S(i) = l\}$. Intuitively, a jurisdiction structure induces a finite partition of the set $[0,1]$ into the Lebesgue measurable sets $N_l^S$ (for $l \in \mathbb{L}$), each of which interpreted as a community located at $l$. We let $n_l^S = \lambda(N_l^S)$ denote the "number of households living at $l$" in the jurisdiction structure $S$. The possibility that $n_l^S = 0$ for some $l$ is, of course, not ruled out. Condition 1) requires the central government to balance its budget. It requires more precisely that the sum of the (possibly negative) grants given to jurisdictions cannot exceed the revenues obtained from taxing households at rate $c$. Condition 2) limits the fiscal power of the central government to raise taxes in a given jurisdiction to the extent of this jurisdiction's tax base. Condition 3) requires tax rates (central and local) and government grants in every location to be such that a household living there consumes non-negative quantities of the public and the private good. We emphasize that negative local tax rates are possible in a world with a central government. A household living in a jurisdiction receiving a large grant may prefer local tax rate to be negative and, therefore, use part of the grant in private spending.

This modeling of the central government covers both the possibility that it transfers money between jurisdictions without taxing households ("horizontal" equalization) and the combination of horizontal and vertical equalization. Yet our model, by restricting central taxation power to linear wealth tax schemes, rules out the possibility of using wealth tax for redistributive purposes (by making it progressive for instance). Of course providing the central government with the *full* power of redistributing wealth (by choosing the tax paid by each household for instance based on its characteristic) would devoid the problem examined in this paper of much of its interest. For any central government that is averse to wealth inequality would obviously choose, if given such a power, to equalize wealth perfectly within a given jurisdiction. But between the full power given to a central government of taxing individually each household, and the extremely small one considered here of taxing all of them at the same rate, there is a large spectrum of possibilities that, undoubtedly, deserve further analysis.

Denote by $\Phi(\tau, \varpi, \omega_i, \gamma, c) = U(\tau\varpi + \gamma, (1-\tau-c)\omega_i)$ the utility received by a household with wealth $\omega_i$ living in a jurisdiction with local tax rate $\tau$, aggregate wealth $\varpi$ and central government grant $\gamma$ when the government chooses a tax rate of $c$. The function $\Phi$ so defined has several properties that we record in the following lemma (whose straightforward proof is omitted).

**Lemma 1** *If $U$ satisfies the conditions discussed above, $\Phi$ is a twice differentiable function of its five arguments, is strictly increasing and concave with respect to $\omega_i$, $\varpi$ and $\gamma$ (taking $\tau$ and $c$ as given) and is strictly concave and single peaked*[3] *with respect to $\tau$ (taking $\omega_i$, $\varpi$, $\gamma$ and $c$ as given).*

One important property of $\Phi$ is its strict single peakedness. It implies that a household with wealth $\omega_i$ facing a central tax rate of $c$ has a unique favorite local tax rate $\tau^*(\varpi, \omega_i, \gamma, c)$ in any jurisdiction with tax base $\varpi$ and central government grant $\gamma$ to which it may belong. This unique favorite local tax rate is the solution of the program:

$$\max_{\tau \in [\frac{-\gamma}{\varpi}, 1-c]} \Phi(\tau, \varpi, \omega_i, \gamma, c) \tag{3}$$

and is, for this reason, a continuous function of all its four arguments.

We are interested in the properties of the likely outcome of a free choice of a location by households in the presence of a central government. This likely outcome must be *stable* in the sense that:

1) each household finds its location optimal given central government's equalization scheme and local tax rates, under the assumption that it can move freely between locations and that it has no effect on jurisdictions' choices of tax rates and aggregate wealth,

2) the central government finds optimal its vector of equalization grants and wealth tax rate, given the partition of individuals into jurisdictions and jurisdictions' choices of local tax rates and,

3) each jurisdiction finds optimal its choice of tax rate and public good provision, given its population, tax base and central government grant.

Concerning the last point, we adopt the view that each jurisdiction's choice of local tax rate is minimally democratic in the sense that it is contained *between* the *smallest* and the *largest* favorite tax rates of the jurisdiction members. The rule for selecting this tax rate is inconsequential for the results. In many models of endogenous jurisdiction formation with public good provision where voting is assumed, the jurisdiction tax rate would the one that occupies the *median position* in the jurisdiction's distribution of favorite taxes. While the analysis of this paper applies to this particular rule of selection of local tax rates, they are valid for other rules as well. Given a jurisdiction structure with a central government $(S, t, g, c)$, define, for every location $l \in \mathbb{L}$ such that $n_l^S > 0$, the smallest and largest favorite tax rates

[3] A function $f : A \to \mathbb{R}$ $(A \subset \mathbb{R})$ is strictly single peaked if, for all $a$, $b$ and $c \in A$ such that $a < b < c$, $f(c) > f(b) \Rightarrow f(b) > f(a)$ and $f(a) > f(b) \Rightarrow f(b) > f(c)$.

$t_{l*}$ and $t_l^*$ respectively by:

$$t_{l*} = \min_{i \in N_l^S} \arg \max_{\tau \in [\frac{-g_l}{\varpi_l}, 1-c]} \Phi(\tau, \varpi_l, \omega_i, g_l, c) \text{ and}$$
$$t_l^* = \max_{i \in N_l^S} \arg \max_{\tau \in [\frac{-g_l}{\varpi_l}, 1-c]} \Phi(\tau, \varpi_l, \omega_i, g_l, c)$$

We shall therefore assume that, in any (non-desert) location $l$, the local tax rate $t_l$ satisfies $t_l \in [t_{l*}, t_l^*]$.

As for the objective of the central government, we represent it, for any function $S : [0,1] \to \mathbb{L}$ and any local tax vector $t \in \mathbb{R}^l$ by means of a social ordering[4] $R$ defined on the set $\mathbb{G}(S,t)$ of all grant vectors $g \in \mathbb{R}^L$ and central tax rate $c \in [0,1]$ such that $g_l + t_l \varpi_l \geq 0$ for every $l \in \mathbb{L}$ and $\sum_{l \in \mathbb{L}} g_l \leq c \int_{[0,1]} \omega_i d\lambda$. For any $(g,c)$ and $(g',c') \in \mathbb{G}(S,t)$, we interpret the statement $(g,c)$ $R$ $(g',c')$ as meaning "equalization scheme $(g,c)$ is weakly better (socially) than equalization scheme $(g',c')$".

With this (for the moment vague) specification of the central government's objective, we define formally stability as follows.

**Definition 1** *Given an economy $(\lambda, \omega, U, \mathbb{L})$, we say that the jurisdiction structure $(S,t,g,c)$ with a central government endowed with a social objective $R$ is stable if*
*1) For every $l$, $l' \in \mathbb{L}$ and all $i \in N_l^S$, $\Phi(t_l, \varpi_l, \omega_i, g_l, c) \geq \Phi(t_{l'}, \varpi_{l'}, \omega_i, g_{l'}, c)$.*
*2) for all $l \in \mathbb{L}$, $t_l \in [t_{l*}, t_l^*]$*
*3) $(g,c)$ $R$ $(g',c')$ for all $(g',c') \in \mathbb{G}(S,t)$.*

This definition of stability rides on the assumption that households as well as local and central governments take their decision simultaneously, considering as given the behavior of others. This assumption generalizes naturally the static Westhoff framework. Yet it may be viewed as restricting unduly the central government's power of shaping the process of jurisdiction formation. An alternative could have been to assume a *two-stage* setting in which the central government would play *before* households and local governments and would choose its redistributive grants and households tax by anticipating the impact of its choice on the stable jurisdiction structure that would emerge in the second stage. While this alternative would certainly be worth exploring in detail, we have chosen to leave it aside in this paper that represents, to the best of our knowledge, the first attempt to introduce a welfarist central government in Tiebout-like models of jurisdiction formation. We simply notice that the two-stage setting for integrating a central government in a Tiebout-like economy involves delicate modeling issues. Let us mention three of them that come to our mind.

[4]An ordering is a reflexive, complete and transitive binary relation.

First, choosing a set of equalization grants and wealth tax rate before knowing the jurisdiction structure that prevail raises the problem of the financial viability of the equalizations grants. What if the central government chooses, in the first stage, an equalization scheme which imposes a tax burden to a jurisdiction which, in the second stage, will be empty ? Second, and more importantly, there may be many stable jurisdiction structures that correspond to a given central government equalization and taxation scheme. If this is the case, how is the central government going to predict which of the stable jurisdiction structures will emerge ? Third, everything else being the same, the central government, at least if it uses a Pareto inclusive social objective, would have a tendency to favour the "trivial" structure in which all households are put in the same jurisdiction and where, therefore, there is no role for a central government. The reason for this tendency is that, the local public good being non-rival, producing any quantity of it in a larger jurisdiction tend to be preferable from a social welfare view point because its cost can be shared by a larger number of tax payers.

We conduct the analysis by considering in turn two specific objectives for the social government: *maxmin* and *generalized utilitarianism*. There are many justifications that could be given to this choice (see e.g. Blackorby, Bossert, and Donaldson (2005) ch. 4) in a classical social choice setting with a finite numbers of households. Yet social choice theory is not very developed for populations with a continuum of individuals (see however Candeal, Chichilnisky, and Induràin (1997)) and, to the best of our knowledge, there are no axiomatic results that would justify maxmin or generalized utilitarian objectives with a continuum of households. We therefore justify this choice by saying that they encompass many welfarist criteria used in the literature.

A maxmin central government compares grant vectors and tax rate by comparing the smallest utility level that these vectors generate. This obviously suppose that utility levels are comparable across households. While the standard definition of the maxmin criterion is provided for economy with a finite (or at least countable) number of households, we must adapt this definition to the continuous setting considered herein. For this sake, given an economy $(\lambda, \omega, U, \mathbb{L})$, define, for every measurable set $I \subset [0,1]$ of households, every public good quantity $Z$ and every non-negative number $a \in \mathbb{R}_+$, the set $\mathbb{U}(Z, a, I) = \{u \in \mathbb{R} : u = U(Z, a\omega_i))$ for some $i \in I\}$. In words $\mathbb{U}(Z, a, I)$ is the (measurable) set of utility numbers achieved by members of $I$ when they all consume $Z$ units of the public good and when their consumption of private good is a linear function of their wealth with slope $a$. Since the utility function used by the government is bounded from below, the set $\mathbb{U}(Z, a, I)$ has a lower bound. With this notation, the maxmin ordering $R^{\max\min}$ used by the central government is defined as follows:

$$(g,c)\ R^{\max\min}\ (g',c') \iff$$
$$\inf_{i\in[0,1]} \bigcup_{l\in\mathbb{L}} \mathbb{U}(t_l\varpi_l + g_l, 1-t_l-c, N_l^S) \quad \geq \quad \inf_{i\in[0,1]} \bigcup_{l\in\mathbb{L}} \mathbb{U}(t_l\varpi_l + g_l', 1-t_l-c', N_l^S)$$

It turns out that there is not much analysis to be performed with such a maxmin central government. For, as established in the following proposition, the only stable jurisdiction structures that can exist with a maxmin central government are those where the jurisdictions' poorest households have the same wealth in all jurisdictions. The trivial jurisdiction structure in which all households are in the same jurisdiction is, of course, a particular example of such jurisdiction structures.

**Proposition 1** *A jurisdiction structure $(S,g,c,t)$ with a maxmin central government is stable iff for all $l$ and $l'$ such that $n_l^S > 0$ and $n_{l'}^S > 0$, $\inf\limits_{i\in N_l^Z} \omega_i = \inf\limits_{h\in N_{l'}^S} \omega_h$*

***Proof.*** *By contraposition, assume that $(S,g,c,t)$ is a stable jurisdiction structure with a welfarist central government in which there are locations $l$ and $l' \in \mathbb{L}$ for which $n_l^S > 0$ and $n_{l'}^S > 0$ such that $\inf\limits_{i\in N_l^Z} \omega_i \neq \inf\limits_{h\in N_{l'}^S} \omega_h$. We wish to show that the welfarist central government can not be maxmin. Without loss of generality, assume that the set $N_l^S$ contains a household who is the worst off in the whole population. Because this household must belong to the set of poorest households in $N_l^S$, one has:*

$$\Phi(t_l, \varpi_l, \inf_{i\in N_l^Z} \omega_i, g_l, c) \leq \Phi(t_k, \varpi_k, \omega_h, g_k, c) \tag{4}$$

*for all $k \in \mathbb{L}$ such that $n_k^S > 0$ and $h \in N_k^S$. By stability one has also:*

$$\Phi(t_l, \varpi_l, \inf_{i\in N_l^Z} \omega_i, g_l, c) \geq \Phi(t_{l'}, \varpi_{l'}, \inf_{i\in N_l^Z} \omega_i, g_{l'}, c) \tag{5}$$

*and:*

$$\Phi(t_{l'}, \varpi_{l'}, \inf_{h\in N_{l'}^Z} \omega_h, g_{l'}, c) \geq \Phi(t_l, \varpi_l, \inf_{h\in N_{l'}^Z} \omega_h, g_l, c) \tag{6}$$

*Either (i) $\inf\limits_{h\in N_{l'}^Z} \omega_h < \inf\limits_{i\in N_l^Z} \omega_i$ or:*
*(ii) $\inf\limits_{h\in N_{l'}^Z} \omega_h > \inf\limits_{i\in N_l^Z} \omega_i$. Yet assuming (i) would imply, using (5) and the fact that $\Phi$ is increasing with respect to private wealth, that:*

$$\Phi(t_l, \varpi_l, \inf_{i\in N_l^Z} \omega_i, g_l, c) > \Phi(t_{l'}, \varpi_{l'}, \inf_{h\in N_{l'}^Z} \omega_h, g_{l'}, c)$$

*in contradiction with (4). Hence (i) can not hold. If (ii) holds, then one has, because of (6) and the monotonicity of* $\Phi$ *with respect to private wealth, that:*

$$\Phi(t_{l'}, \varpi_{l'}, \inf_{h \in N_{l'}^Z} \omega_h, g_{l'}, c) > \Phi(t_l, \varpi_l, \inf_{i \in N_l^Z} \omega_i, g_l, c)$$

*so that the household whose income is (arbitrarily close to)* $\inf_{i \in N_l^Z} \omega_i$ *is strictly worse off than the poorest household in jurisdiction* $l'$*. In that case, let* $W$ *be defined by:*
$W = \{l'' \in \mathbb{L} : \Phi(t_{l''}, \varpi_{l''}, \inf_{i \in N_{l''}^Z} \omega_i, g_{l''}, c) = \Phi(t_l, \varpi_l, \inf_{i \in N_l^Z} \omega_i, g_l, c)\}$*. The set* $W$ *is clearly non-empty since* $l \in W$*. Consider then taking away from jurisdiction* $l'$ *some amount of grant* $\Delta$ *and dividing it up equally among all jurisdictions in* $W$ *so as to keep constant the central government budget constraint. For a suitably small* $\Delta$*, this change in the central government transfer policy increases the well-being of all households in the jurisdictions contained in* $W$ *(including the worst off) while keeping the well-being of the worst off household in jurisdiction* $l'$ *above. This shows that the original equalization grant vector* $g$ *was not maximizing a maxmin ordering, given* $c$*.*
■

The intuition behind this result is quite simple. A maxmin government wants to transfer money to the jurisdiction that contains one of the worst off households (who must clearly be the poorest in its jurisdiction). By stability, any such worst off household prefers staying in its jurisdiction than moving elsewhere while the households in other jurisdictions - including the poorest in them - also prefer staying where they are than moving to the jurisdiction containing the worst off households. Except if the wealth of these poorest households in all jurisdictions is the same, these two conditions for stability imply that worst-off households are strictly worse off than at least one household who is the poorest in its jurisdiction. But if this is the case, then the transfers given by the central government to jurisdictions are not optimal from a maxmin point of view. Notice that the reasoning holds irrespective of the tax rate chosen by the central government.

Jurisdiction structures in which all non-desert jurisdictions contain some of the population poorest households are, admittedly, rather peculiar. For one thing, in any such jurisdiction structure, a maxmin central government would choose not to intervene at all and will therefore not be empirically observable. Hence, if one wants to go beyond the message that, except for this particular case, no analysis of stable jurisdiction structures can be performed in presence of a central government, we must abandon the assumption that its objective is the maxmin criterion. We do that by assuming instead that it uses a generalized-utilitarian criterion.

This criterion compares any two combinations $(g, c)$ and $(g', c')$ of equalization grants and central tax rate by the ordering $R^{GU}$ defined by:

$$(g, c)\ R^U\ (g', c') \iff$$

$$\sum_{l \in \mathbb{L}} \int_{N_l^S} \Psi(\Phi(t_l, \varpi_l, \omega_i, g_l, c)) d\lambda \quad \geq \quad \sum_{l \in \mathbb{L}} \int_{N_l^S} \Psi(\Phi(t_l, \varpi_l, \omega_i, g'_l, c')) d\lambda$$

where $\Psi : \mathbb{R} \to \mathbb{R}$ is a function that evaluates individual utility functions from a social view-point. The Pareto principle requires $\Psi$ to be increasing while utility-inequality aversion considerations suggest that $\Psi$ be concave. We will also assume that $\Psi$ is differentiable. A well-known example of such a generalized utilitarian criterion is the global symmetric mean of order $r$ (for some real number $r \leq 1$) in which $\Psi$ is defined by:

$$\begin{aligned} \Psi(u) &= u^r \text{ if } r \in ]0, 1] \\ &= \ln u \text{ if } r = 0 \\ &= -u^r \text{ if } r < 0 \end{aligned}$$

This family contains the standard utilitarian criterion (for $r = 1$) and approaches the maxmin criterion (as $r$ approaches $-\infty$). An axiomatic characterization of the family of generalized-utilitarian criteria is provided in Blackorby, Bossert, and Donaldson (2005) (theorem 4.7) in a setting with a finite population.

For a general utilitarian government, there are several stable jurisdiction structures that are not of the very specific variety described in proposition 1, as illustrated in the following simple example (which uses conventional utilitarianism).

**Example 1** *There is a mass 7 of households with utility function* $U(Z, x) = \ln(1+Z)+x$. *There is a mass 3 of households with a wealth of* $2$ *and a mass 4 of households with wealth of* $3/2$. *Consider the 2-jurisdictions structure in which the central government gives no equalization grants (and therefore levy no taxes) and where the households with wealth 2 are all put in jurisdiction 1 while the others are put in jurisdiction 2. The local tax rates* $t_1$ *and* $t_2$ *that prevail in the two jurisdictions are the favorite ones of the identical households who live there. Solving program (3), these optimal tax rates are easily found to be* $t_1 = 1/3$ *and* $t_2 = 1/2$. *We notice that with these tax rates, any household in jurisdiction 1 enjoys a utility level of* $\ln 3 + 4/3 \approx 2.4319$, *which is larger than the utility of* $\ln 4 + 1 \approx 2.386$ *it would enjoy if it were to move to jurisdiction 2 and to get the tax and public good package available there. Analogously, a household in jurisdiction 2 enjoys a utility level of* $\ln 4 + 3/4 \approx 2.1363$ *by staying where it is compared to a (lower) utility of* $\ln 3 + 1 \approx 2.0986$ *if it were to move. Hence households have no incentive to move from their jurisdiction. To see that a utilitarian central* *government finds optimal to gives zero equalization grants and raise no taxes, it is sufficient (given concavity of $U$) to show that:*

$$(0,0) \in \arg\max_{\gamma,t} 3[\ln(3+\gamma) + 2(2/3-t) + 4[\ln(4+12t-\gamma) + \frac{3}{2}(1/2-t)]$$

*so that the (1st order) conditions:*

$$\frac{3}{(3+\gamma^*)} = \frac{4}{(4+12t^*-\gamma^*)} \tag{7}$$

*and:*

$$-6 + \frac{48}{(4+12t^*-\gamma^*)} - 6 = 0 \tag{8}$$

*hold for $\gamma^* = t^* = 0$, which is indeed the case. Hence we have a non trivial stable structure with a central government.*

In order to identify the condition on households preferences that is necessary and sufficient for guaranteeing that wealth-segregation of any stable jurisdiction structure, one needs first to define what is meant by a wealth-segregation. We take the definition to be that used in Gravel and Thoron (2007) (see also Westhoff (1977) and Greenberg and Weber (1986)).

**Definition 2** *A jurisdiction structure with a central government $(S, g, c; t)$ is wealth-segregated if, for every locations $l$, $l' \in \mathbb{L}$ for which $n_l^S > 0$ and $n_{l'}^S$ and every households $h$, $i$ and $k \in [0,1]$ such that $h < i < k$, $h$, $i \in N_l^S$ and $i \in N_{l'}^S \Rightarrow t_l = t_{l'} = \frac{g_{l'} - g_l}{\overline{\omega}_{l'} - \overline{\omega}_l}$.*

In words, a jurisdiction structure is wealth-stratified if, whenever a (non-null) jurisdiction contains two households $h$ and $k$ with different levels of wealth, it also contains all households whose wealth levels are strictly between that of $h$ and $k$ or, if it does not contain those households, it is because they belong to some (non-null) jurisdiction that offers the same tax rate and the same amount of public good than $j$.

# 3 RESULTS

As in Gravel and Thoron (2007), the monotonicity of $\tau^*$ with respect to household's wealth (given jurisdiction's wealth and central government grant) will be a key element for guaranteeing the wealth segregation of stable jurisdiction structures. This property of monotonicity of the household's most preferred tax rate can be expressed conveniently in terms of standard consumer theory. In order to do this, we establish the following lemma whose proof, similar to that of lemma 2 in Gravel and Thoron (2007), is omitted.

**Lemma 2** *Let* $(\overline{\omega}, \omega_i, \gamma, c) \in \mathbb{R}^2_{++} \times \mathbb{R} \times [0,1]$. *Then for all* $U$ *satisfying the above properties,* $\frac{1}{\overline{\omega}}[Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma)$ *is the solution of (3).*

Lemma 2 states that, in a jurisdiction with aggregate wealth $\varpi$ and central government transfer $\gamma$, the favorite tax rate of a household with (net of central government tax) wealth $\omega_i(1-c)$ can be viewed as the *expenditure* that the household would like to devote to local public good in excess of the central government grant if the prices of public and the private goods were $\frac{1}{\overline{\omega}}$ and $\frac{1}{\omega_i}$, and if this household had an income of $1 - c + \frac{\gamma}{\overline{\omega}}$. Interpreted in this fashion, monotonicity of $\tau^*$ with respect to $\omega_i$ is *equivalent* to monotonicity of the Marshallian demand for public good with respect to the price of the private good. In the language of standard consumer theory, this is equivalent to requiring the public good to be, at any price of public good, either always a gross *complement* to (if $Z^M$ is monotonically *decreasing* with respect to $p_x$) or always a gross *substitute* for (if $Z^M$ is monotonically *increasing* with respect to $p_x$) the private good. We state formally this state of affairs, proved in Gravel and Thoron (2007), using the regularity condition 1.

**Lemma 3** *For every* $U \in \mathbb{U}$*, the function* $\tau^*$ *that solves (3) is monotonic with respect to* $\omega_i$ *for any given jurisdiction level* $\overline{\omega}$ *and per capita wealth if and only if the public good is always either a gross complement to, or a gross substitute for, the private good.*

We refer to this property according to which the substitutability/ complementarity relationship between the public and private good is independent from all possible prices as to the **Gross Substitutability/ Complementarity (GSC) condition**. Although not unreasonable, the GSC condition is nonetheless a significant restriction that, as discussed in Gravel and Thoron (2007), can be violated even by additively separable preferences.

An information used in Gravel and Thoron (2007) to show that the GSC condition is necessary and sufficient for guaranteeing the segregation of any stable jurisdiction structure is the structure of households' indifference curves in the tax-jurisdiction's wealth space. While this information is also useful in the present context, we need to account for the fact that the relevant space of location characteristics is now three, rather than two, dimensional and must include central government grant as well as local tax rate and jurisdiction's aggregate wealth. Specifically, the indifference surface of a household with (net of central government) wealth $(1-c)\omega_i$ passing through some point $(\overline{\tau}, \overline{\omega}, \overline{\gamma}) \in \mathbb{R}^3$ such that $\Phi(\overline{\tau}, \overline{\omega}, \omega_i, \overline{\gamma}, c) = \overline{\Phi}$ is the graph of the implicit function $f^{\overline{\Phi}} : [\frac{-\overline{\gamma}}{\overline{\omega}}, 1] \times \mathbb{R} \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ defined by $\Phi(\tau, f^{\overline{\Phi}}(\tau, \gamma, c; \omega_i), \omega_i, \gamma, c) \equiv \overline{\Phi}$. The assumption imposed on $U$ guarantees that the function $f^{\overline{\Phi}}$ exists and is derivable everywhere. Its partial derivative

$f_{\tau}^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$ with respect to $\tau$ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f_{\tau}^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{1}{\overline{\tau}}\left[\frac{\omega_i}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_i)} - \varpi\right] \quad (9)$$

where $\varpi = f^{\Phi}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$. Figure 1 below illustrates the shape of these indifference curves in the $(\tau, \varpi)$ plane for given values of $\overline{\gamma}$ and $c$. Specifically, indifference curves of a household with private wealth $(1-c)\omega_i$ are $U$-shaped and reach a minimum at this household's most preferred tax rate for the corresponding jurisdiction wealth level. It can be seen indeed that, at the minimum of an indifference curve, the term within the bracket of (9) is zero thanks to the first order conditions of (3)). Despite what figure 1 suggests, indifference curves need *not* be globally convex. The only property that indifference curves possess is that of being "single-through" (monotonically decreasing at the left of the minimum and monotonically increasing at the right).

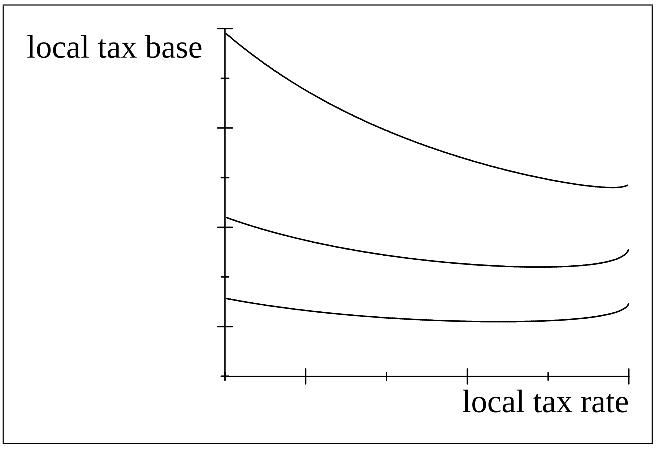


Figure 1

Analogously, one can fix local tax rate at $\overline{\tau}$ and examine the property of the derivative of $f^{\overline{\Phi}}$ with respect to the central government's grant. This partial derivative $f_{\gamma}^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i, )$ with respect to $\gamma$ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f_{\gamma}^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{-1}{\overline{\tau}} \quad (10)$$

Hence, when looked in the $(\gamma, \overline{\omega})$ space, indifference surfaces are straight line with negative slope (if at least $\overline{\tau}$ is positive). There is therefore a constant marginal trade off between tax base and central government grant as envisaged by a mobile household. This is of course not surprising since both central government grant and local tax base are perfectly substitutable ways of getting public expenditure in a given jurisdiction. The rate at which the household is willing to sacrifice local tax base in order to get more central government transfer depends obviously upon the local tax rate that converts tax base into public spending. Figure 2 shows a typical indifference surface in the tax rate, tax base and central government space in which location choice by households is made.

We first establish, in the following lemma, that the ordering of the *slopes* of these indifference curves at every point in the tax- jurisdictions wealth space (for a given level of central government grant) coincides with the ordering of the households' wealth if and only if preferences for the public and the private good satisfy the GSC condition.

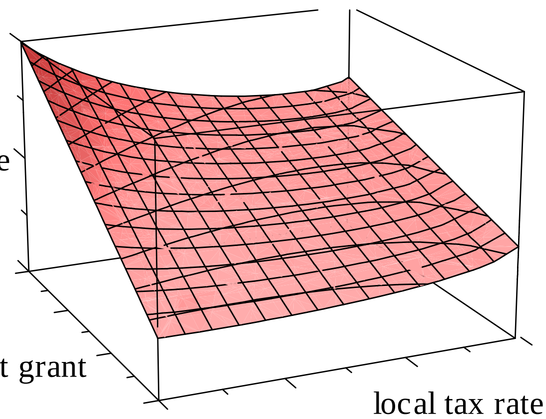


Figure 2

**Lemma 4** *Assume that households preferences are represented by a utility function satisfying the properties mentioned above. Then, $Z^M$ is everywhere a gross substitute (resp. complement) to the private good if and only if one has, at any $(\overline{\tau}, \varpi, \overline{\gamma}, c) \in \mathbb{R}^4$ satisfying $\overline{\gamma} \geq -\overline{\tau}\varpi$, $\overline{\tau} \leq 1 - c$, $\varpi \geq 0$ and $c \in [0, 1]$, $f_{\tau}^{\overline{\Phi}_j}(\overline{\tau}, \overline{\gamma}; \omega_h) \leq$ (resp. $\geq$) $f_{\tau}^{\overline{\Phi}_k}(\overline{\tau}, \overline{\gamma}, \omega_k)$ for every $h$, $k$ such that $\omega_h < \omega_k$ where, for every $i$, $\overline{\Phi}_i = \Phi(\overline{\tau}, \varpi, \overline{\gamma}, c; \omega_i)$.*

***Proof.*** *We provide the argument for the case of gross substitutability (the complementarity case being symmetric). For the first implication, assume that $Z^M$ is everywhere increasing with respect to $p_x$ and let $(\overline{\tau}, \varpi, \overline{\gamma}, c) \in \mathbb{R}^4$ be a combination of local tax rate $\overline{\tau}$, jurisdiction wealth $\varpi$, central government grant $\overline{\gamma}$ and central government tax rate $c$ satisfying $\overline{\gamma} \geq -\overline{\tau}\varpi$,$\overline{\tau} \leq 1 - c$, $\varpi \geq 0$ and $c \in [0, 1]$ and let $h$ and $k$ be two households such that $\omega_h < \omega_k$. Refer to figure 3 and define $\overline{\omega}(h)$ and $\omega_i(h)$ to be the numbers that generate public and private good prices $1/\overline{\omega}(h)$ and $1/\omega_i(h)$ which would lead a consumer with an income of $1 - c + \frac{\overline{\gamma}}{\overline{\omega}(h)}$ to choose the bundle $(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_h)$ of public and private good. Hence $\overline{\omega}(h)$ and $\omega_i(h)$*

*satisfy the standard tangency and budget equality conditions:*

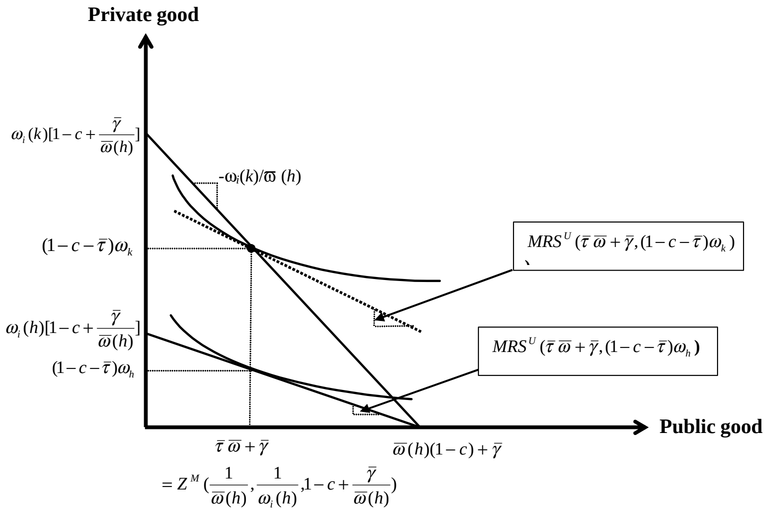


*Figure 3.*

$$MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1-c-\overline{\tau})\omega_h) = \frac{\omega_i(h)}{\overline{\omega}(h)}$$

*and*

$$\frac{\overline{\tau}\varpi + \overline{\gamma}}{\overline{\omega}(h)} + \frac{(1-c-\overline{\tau})\omega_h}{\omega_i(h)} = 1 - c + \frac{\overline{\gamma}}{\overline{\omega}(h)}$$

*Combining these two equations yields:*

$$\frac{(1-c-\overline{\tau})}{\overline{\omega}(h)(1-c) - \overline{\tau\omega}} = \frac{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1-c-\overline{\tau})\omega_h)}{\omega_h} \tag{11}$$

*Define now $\omega_i(k)$ to be a level of household wealth which would generate private good price $1/\omega_i(k)$) that is just sufficient to enable a consumer with the same income of $1-c+\frac{\overline{\gamma}}{\overline{\omega}(h)}$ and facing public good price $1/\overline{\omega}(h)$ to afford the bundle $(\overline{\tau\omega} + \overline{\gamma}, (1-c-\overline{\tau})\omega_k)$. This $\omega_i(k)$ (which is clearly larger than $\omega_i(h)$ if $\omega_k > \omega_h$) is defined by the budget constraint equality:*

$$\begin{aligned} \frac{\overline{\tau}\varpi + \overline{\gamma}}{\overline{\omega}(h)} + \frac{(1-c-\overline{\tau})\omega_k}{\omega_i(k)} &= 1 - c + \frac{\overline{\gamma}}{\overline{\omega}(h)} \\ &\Longleftrightarrow \\ \omega_i(k) &= \frac{\overline{\omega}(h)(1-c-\overline{\tau})\omega_k}{\overline{\omega}(h)(1-c) - \overline{\tau}\varpi} \end{aligned} \tag{12}$$

*Now, since $Z^M$ is increasing with respect to $p_x$, we must have $Z^M(\frac{1}{\overline{\omega}(h)}, \frac{1}{\omega_i(h))}, 1-c+\frac{\overline{\gamma}}{\overline{\omega}(h)}) \geq Z^M(\frac{1}{\overline{\omega}(h)}, \frac{1}{\omega_i(k)}, 1-c+\frac{\overline{\gamma}}{\overline{\omega}(h)})$ and, therefore (see figure 3), the* *slope of the indifference curve passing through $(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-c-\overline{\tau})\omega_k)$ must be, in absolute value, less than the price ratio $\omega_i(k)/\overline{\omega}(h)$. Formally, this amounts to saying that:*

$$MRS^U(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-c-\overline{\gamma})\omega_k) \leq \frac{\omega_i(k)}{\overline{\omega}(h)}$$

*or, using (12):*

$$\frac{MRS^U(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-\overline{\tau})\omega_k)}{\omega_k} \leq \frac{(1-c-\overline{\tau})}{\overline{\omega}(h)(1-c)-\overline{t}\overline{\omega}} \tag{13}$$

*Combining inequality (13) and equality (11), we get:*

$$\begin{array}{ccc} \dfrac{MRS^U(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-c-\overline{\tau})\omega_k)}{\omega_k} & \leq & \dfrac{MRS^U(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-c-\overline{\tau})\omega_h)}{\omega_h} \\ & \Longleftrightarrow & \\ \dfrac{\omega_k}{MRS^U(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-c-\overline{\tau})\omega_k)} & \geq & \dfrac{\omega_h}{MRS^U(\overline{\tau}\overline{\omega}+\overline{\gamma},(1-c-\overline{\tau})\omega_h)} \end{array}$$

*which, using the definition of $f_\tau^{\overline{\Phi}_j}$ provided by (9), establishes the result. For the second implication, assume that $Z^M$ is not everywhere increasing with respect to $p_x$ so that there are positive private good prices $p_x^0$, $p_x^1$ and $p_x^2$ satisfying $p_x^0 < p_x^1 < p_x^2$ as well as positive public good price $p_Z$ and income level $R$ for which one has:*

$$Z^M(p_Z,p_x^0,R) = Z^M(p_Z,p_x^2,R) > Z^M(p_Z,p_x^1,R) \tag{14}$$

*Let $c \in [0,1]$ be any central government tax rate and let $\varpi$, $\omega_i$ and $\overline{\gamma}$ be defined by:*

$$\varpi = \frac{1}{p_Z} \tag{15}$$

$$\omega_i = \frac{1}{p_x^i} \text{ and} \tag{16}$$

$$\overline{\gamma} = \varpi(R-1+c) \tag{17}$$

*for $i=0,1,2$. By lemma 2 and definitions (15)-(17), one has $Z^M(p_Z,p_x^i,R) = \tau^*(\varpi,\omega_i,\overline{\gamma},c)\varpi+\overline{\gamma}$ and $x^M(p_Z,p_x^i,R) = (1-c-\tau^*(\varpi,\omega_i,\overline{\gamma},c)\omega_i)$ for $i=0,1,2$. Consider the tax rate $\overline{\tau}$ such that:*

$$\overline{\tau} = \frac{Z^M(p_Z,p_x^0,R)-\overline{\gamma}}{\varpi} = \frac{Z^M(p_Z,p_x^2,R)-\overline{\gamma}}{\varpi} \tag{18}$$

*Using (9), we have:*

$$\begin{aligned} f_\tau^{\overline{\Phi}}(\overline{\tau},\overline{\gamma},c;\omega_0) &= \frac{1}{\overline{\tau}}[\frac{\omega_0}{MRS(\overline{\tau}\varpi+\overline{\gamma},(1-c-\overline{\tau})\omega_0)}-\varpi] \\ &= 0 \text{ (by the FOCs of the consumer's program)} \\ &= \frac{1}{\overline{\tau}}[\frac{\omega_2}{MRS(\overline{\tau}\varpi+\overline{\gamma},(1-c-\overline{\tau})\omega_2)}-\varpi] \\ &= f_\tau^{\overline{\Phi}}(\overline{\tau},\overline{\gamma},c;\omega_2) \end{aligned}$$

*Moreover, since*

$$\begin{aligned} Z^M(p_Z, p_x^1, R) &= \tau^*(\varpi, \omega_1, \gamma, c)\varpi + \overline{\gamma} \\ &< Z^M(p_Z, p_x^2, R) \\ &= \tau^*(\varpi, \omega_2, \overline{\gamma}, c)\varpi + \overline{\gamma} \\ &= \tau^*(\varpi, \omega_0, \overline{\gamma}, c)\varpi + \overline{\gamma} \\ &= Z^M(p_Z, p_x^0, R) \end{aligned}$$

*one has* $MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_1) < \frac{\omega_1}{\varpi}$*. Hence thanks to (9), one has*

$$\begin{aligned} f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c, \omega_1) &= \frac{1}{\overline{\tau}}[\frac{\omega_1}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_1)} - \varpi] \\ &> 0 \end{aligned}$$

*It follows therefore that* $f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_0) = f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_2) < f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c, \omega_1)$ *for* $\omega_0$*,* $\omega_1$ *and* $\omega_2$ *satisfying* $\omega_0 > \omega_1 > \omega_2$ *so that the slopes of indifference curves in the* $(\tau, \varpi)$ *plane for a certain* $\overline{\gamma}$ *and* $c$ *are not monotonically decreasing with respect to* $\omega_i$ *.* ■

This lemma says that the GSC condition is equivalent to the requirement that the slope of the indifference surfaces in the local tax rate and jurisdiction wealth space be *monotonic* with respect to private wealth at any point of that space. In proposition 3 below, we shall show that this monotonicity of the slopes in the two-dimensional space of tax rate and aggregate wealth for a given government grant holds true as well in the three dimensional space of government grants, local tax and jurisdiction's wealth.

We now establish that, if preferences are additively separable, and if the utility function aggregated by the generalized-utilitarian central government is the additively separable numerical representation of those preferences, then the GSC condition is *necessary* for the wealth segregation of any stable jurisdiction structure.

**Proposition 2** *Assume preferences are additively separable. Then, a stable jurisdiction structure with a generalized-utilitarian central government who uses the additively separable numerical representation of these preferences as utility function is segregated only if the preferences satisfies the GSC condition.*

***Proof.*** *Assume that the GSC condition is violated. Then there are private good prices* $p_x^0$*,* $p_x^1$ *and* $p_x^1$ *satisfying* $0 < p_x^0 < p_x^1 < p_x^2$ *and public good price* $p_Z > 0$ *such that:*

$$\begin{aligned} Z^M(p_Z, p_x^0, 1) &= Z^M(p_Z, p_x^2, 1) \\ &> Z^M(p_Z, p_x^1, 1) \text{ or} \\ &< Z^M(p_Z, p_x^1, 1) \end{aligned}$$

*Our objective is to show the existence of an economy where a non-segregated stable jurisdiction structure can be constructed. We provide the construction by assuming the first of these two inequalities, the argument being symmetric for the second. Define the strictly positive numbers $\theta$, $\alpha$, $\beta$ and $\delta$ by:*

$$\begin{aligned}\theta &= \frac{1}{p_Z}\\ \alpha &= \frac{1}{p_x^2}\\ \beta &= \frac{1}{p_x^1}\\ \delta &= \frac{1}{p_x^0}\end{aligned}$$

*Consider now an economy where a mass $n_\alpha$ of households have wealth $\alpha$, a mass $n_\beta$ of households have wealth $\beta$ and a mass $n_\delta$ of households have wealth $\delta$. In order to construct a stable jurisdiction structure that is not segregated, we are going to put households with wealth $\alpha$ and $\delta$ in one jurisdiction - called it 1- and households with wealth $\beta$ in another, 2 say. For this purpose we are going to choose positive numbers of households $n_\alpha$, $n_\beta$ and $n_\delta$ so that:*

$$n_\alpha\alpha + n_\delta\delta = n_\beta\beta = \theta \tag{19}$$

*There is clearly no difficulty, given any $\alpha$, $\beta$, $\delta$ and $\theta$ satisfying the properties above, of finding positive numbers satisfying (19). Without a central government, such a jurisdiction structure, that is clearly non-segregated, would be stable because the two jurisdictions created would have the same tax base $\theta$ and each household will get in its jurisdiction of residence its favorite local tax rate given $\theta$. With a central government however, this jurisdiction structure can be stable only if the central government finds optimal, given the two jurisdictions and local tax rates, to perform no equalization grants and to raise no taxes. Hence, we must prove that positive numbers $n_\alpha$, $n_\beta$ and $n_\delta$ can be found in such a way that a generalized-utilitarian central government who uses the additively separable numerical representation of households' preference provided by (1) would find optimal to give no equalization grants to jurisdictions and to collect no taxes as a result. That is to say, we must prove that positive numbers $n_\alpha$, $n_\beta$ and $n_\delta$ such that:*

$$\begin{aligned}(0,0) \in \arg\max_{(\gamma,c)\in\mathbb{R}\times[0,1]} & n_\alpha\Psi(f(Z^M(p_Z,p_x^2,1)+\gamma)+h(x^M(p_Z,p_x^2,1)-c\alpha))\\ &+n_\beta\Psi(f(Z^M(p_Z,p_x^1,1)+2\theta c-\gamma)+h(x^M(p_Z,p_x^2,1)-c\beta))\\ &+n_\delta\Psi(f(Z^M(p_Z,p_x^0,1)+\gamma)+h(x^M(p_Z,p_x^0,1)-c\delta))\end{aligned} \tag{20}$$

*can be found. Since the objective function of the program (20) is concave, any combination of values of jurisdiction 1's grant $\gamma^*$ and wealth tax $c^*$ that* *satisfy the first order conditions of this program is a solution to it. Using the (assumed) fact that $Z^M(p_Z, p_x^2, 1) = Z^M(p_Z, p_x^0, 1)$, the first order conditions of (20) for $\gamma^* = 0$ and $c^* = 0$ write:*

$$(n_\alpha \Psi^{2\prime} + n_\delta \Psi^{0\prime}) \frac{\partial f(Z^M(p_Z, p_x^2, 1))}{\partial Z} = n_\beta \Psi^{1\prime} \frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z} \tag{21}$$

*and:*

$$n_\alpha \alpha \Psi^{2\prime} \frac{\partial h(x^M(p_Z, p_x^2, 1))}{\partial x} + n_\beta \beta \Psi^{1\prime} \frac{\partial h(x^M(p_Z, p_x^1, 1))}{\partial x} + n_\delta \delta \Psi^{0\prime} \frac{\partial h(x^M(p_Z, p_x^0, 1))}{\partial x}$$

$$= 2\theta n_\beta \Psi^{1\prime} \frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z} \tag{22}$$

*where, for $k = 0,\ 1,\ 2$, $\Psi^{k\prime} > 0$ denotes the derivative of $\Psi$ evaluated at $f(Z^M(p_Z, p_x^k, 1)) + h(x^M(p_Z, p_x^k, 1))$. We notice that, thanks to the decreasing monotonicity of the indirect utility function with respect to prices $\Psi(Z^M(p_Z, p_x^2, 1) + h(x^M(p_Z, p_x^2, 1)) < \Psi(Z^M(p_Z, p_x^1, 1) + h(x^M(p_Z, p_x^1, 1)) < \Psi(Z^M(p_Z, p_x^0, 1) + h(x^M(p_Z, p_x^0, 1))$. Since $\Psi$ is concave, one has therefore:*

$$\Psi^{2\prime} > \Psi^{1\prime} > \Psi^{0\prime} > 0 \tag{23}$$

*By definition of the Marshallian demands, condition (22) writes (thanks to additive separability):*

$$n_\alpha \theta \Psi^{2\prime} \frac{\partial f(Z^M(p_Z, p_x^2, 1))}{\partial Z} + n_\beta \theta \Psi^{1\prime} \frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z} + n_\delta \theta \Psi^{0\prime} \frac{\partial f(Z^M(p_Z, p_x^0, 1))}{\partial Z}$$

$$= 2\theta n_\beta \Psi^{1\prime} \frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z}$$

*or (since $Z^M(p_Z, p_x^2, 1) = Z^M(p_Z, p_x^0, 1)$):*

$$(n_\alpha \Psi^{2\prime} + n_\delta \Psi^{0\prime}) \frac{\partial f(Z^M(p_Z, p_x^2, 1))}{\partial Z} = n_\beta \Psi^{1\prime} \frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z}$$

*which is nothing else than condition (21). Hence, the only other condition that the numbers $n_\alpha$, $n_\beta$ and $n_\delta$ must satisfy beside (19) is condition (21). Since $\alpha$, $\beta$, $\delta$ and $\theta$ are given positive numbers, we have that $n_\beta = \theta/\beta > 0$. Hence, in order for the proposed jurisdiction structure to be stable with a generalized utilitarian government who chooses (optimally) not to intervene, we only need to find positive numbers $n_\alpha$ and $n_\delta$ such that equalities:*

$$n_\alpha = \frac{\theta}{\alpha} - \frac{\delta}{\alpha} n_\delta \tag{24}$$

*and:*

$$n_\alpha = \frac{\Psi^{1\prime} \theta \partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{2\prime} \beta \partial f(Z^M(p_Z, p_x^2, 1))/\partial Z} - \frac{\Psi^{0\prime}}{\Psi^{2\prime}} n_\delta$$

*hold. From the first order condition that defines Marshallian demands, we can write the later equality (using additive separability) as: :*

$$n_\alpha = \frac{\theta\Psi^{1\prime}\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\alpha\Psi^{2\prime}\partial h(x^M(p_Z, p_x^2, 1))/\partial x} - \frac{\Psi^{0\prime}}{\Psi^{2\prime}} n_\delta \tag{25}$$

*Since the preferences represented by the utility function are additively separable, no good is inferior and, as a result, the private good is not a Giffen good. Hence $x^M$ is decreasing with respect to its own price so that, since $p_x^1 < p_x^2$, $x^M(p_Z, p_x^1, 1) > x^M(p_Z, p_x^2, 1)$ and, since h is concave, $\partial h(x^M(p_Z, p_x^1, 1))/\partial x < \partial h(x^M(p_Z, p_x^2, 1))/\partial x$. Since by (23), $\Psi^{1\prime} < \Psi^{2\prime}$, we conclude that the intercept of the linear equation (25) is smaller than that of equation (24). Moreover, the abscissa at the origin, denoted $n_\delta^0(1)$, of the linear equation (24) is given by:*

$$n_\delta^0(1) = \frac{\theta}{\delta}$$

*while the abscissa at the origin $n_\delta^0(2)$ of equation (25) is:*

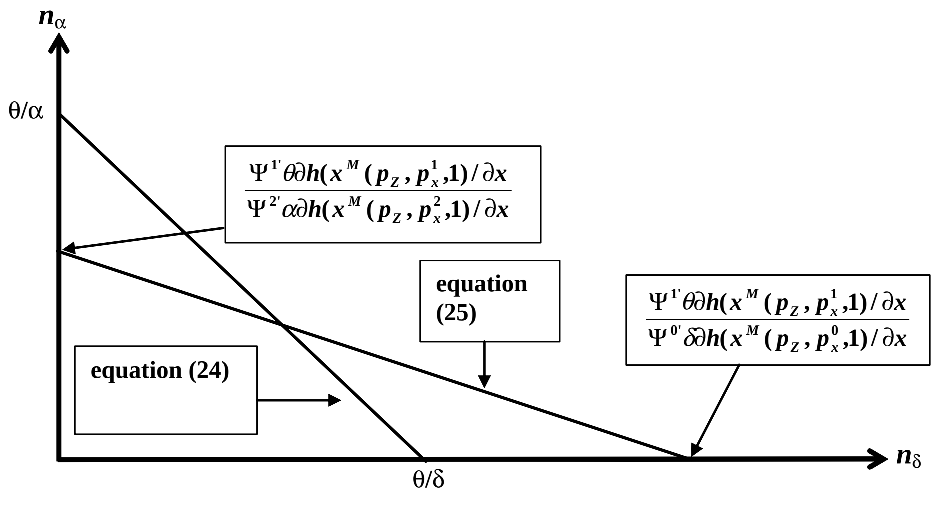


*Figure 4.*

$$\begin{aligned}
n_\delta^0(2) &= \frac{\Psi^{1\prime}\theta\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\Psi^{0\prime}\alpha\partial h(x^M(p_Z, p_x^2, 1))/\partial x} \\
&= \frac{\Psi^{1\prime}\theta\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{0\prime}\beta\partial f(Z^M(p_Z, p_x^2, 1))/\partial Z} \text{ (by separability and the definition of Marshallian demands)} \\
&= \frac{\Psi^{1\prime}\theta\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{0\prime}\beta\partial f(Z^M(p_Z, p_x^0, 1))/\partial Z} \text{ (since } Z^M(p_Z, p_x^0, 1) = Z^M(p_Z, p_x^2, 1)\text{)} \\
&= \frac{\Psi^{1\prime}\theta\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\Psi^{0\prime}\delta\partial h(x^M(p_Z, p_x^0, 1))/\partial x} \text{ (by separability and the definition of Marshallian demands)}
\end{aligned}$$

*Now, again, since the private good is not inferior (and therefore not Giffen) we obtain, using concavity of h and inequality (23), that* $n_\delta^0(2) > n_\delta^0(1) > 0$*. Hence, the two straight lines represented by equations (24) and (25) are as in figure 4) and cross in the strictly positive orthant. This shows the existence of positive numbers* $n_\alpha$ *and* $n_\delta$ *satisfying equations (24) and (25) and this completes the proof.* ■

It is worth mentioning that the proof of this proposition is somewhat different than the one provided in Gravel and Thoron (2007).The reason for the difference is the additional constraint imposed by the optimality of the central government (non) intervention in the construction of the stable but non-segregated jurisdiction structure for any violation of the GSC condition. This constraint obviously increases the difficulty of the construction of the economy giving rise to such a stable non-segregated jurisdiction structure. The additive separability condition plays a key role in this construction and we do not know whether we could obtain the construction without such an assumption. We emphasize, however, that proposition 2 proves in fact something slightly stronger than what is required. Indeed, what is established in proposition 2 is that, for any violation of the GSC condition obtained with additively separable preferences, one can find a non-segregated stable jurisdiction structure in which a generalized utilitarian government finds optimal to perform zero equalization (and accordingly to levy no wealth taxes). The possibility of proving, less demandingly, that any violation of the GSC condition can give rise to a non-segregated stable jurisdiction structure in which the utilitarian central government performs non-zero equalization without assuming additive separability of households utility function remains an open, if not difficult, question.

We notice also that the assumption that there is a continuum of households plays a role in this proof. Specifically, the proof establishes, for any violation of the GSC condition, the existence of strictly positive numbers of households that can be put in a non-segregated but yet stable jurisdiction structure. Yet, we are not capable of proving, for any violation of the GSC condition, the existence of integral numbers of households that can be put in a stable but non-segregated jurisdiction structure.

We now establish, without any further condition on household's preferences, the converse proposition that the GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure. No additive separability and, actually, no continuum hypothesis is needed for this proposition.

**Proposition 3** *Assume that households' preferences satisfy the GSC condition. Then, any stable jurisdiction structure with a Utilitarian central government is wealth segregated.*

***Proof.*** *We sketch the argument for the case where public good is everywhere a gross complement to the private good. Assume therefore that* $Z^M$ *is decreasing with respect to* $p_x$ *and, by contradiction, let* $(S, g, c, t)$ *be a jurisdiction structure that is not wealth-stratified. Hence, there are jurisdictions* $l$ *and* $l' \in \{1, ..., l\}$ *(with* $l \neq l'$, $n_l^S > 0$ *and* $n_{l'}^S > 0$*), and households* $h$, $i$ *and* $k \in [0, 1]$ *with* $h < i < k$ *for which one has* $h$ *and* $k \in N_l^S$, $i \in N_{l''}^S$ *and either* $t_l \neq t_{l'}$ *or* $t_l \varpi_l + g_l \neq t_{l'} \varpi_{l'} + g_{l'}$*. It is clear that if only one of the two inequalities* $t_l \neq t_{l'}$ *and* $t_l \varpi_l + g_l \neq t_{l'} \varpi_l + g_{l'}$ *holds, then the jurisdiction structure can not be stable because there would be unanimity of the inhabitants of one of the jurisdictions* $l$ *and* $l'$ *to go to the jurisdiction with the low tax rate (if* $t_l \neq t_{l'}$ *and* $t_l \varpi_l + g_l = t_{l'} \omega_{l'} + g_{l'}$*) or to the jurisdiction with the largest public good provision (if* $t_l = t_{l'}$ *and* $t_l \varpi_l + g_l \neq t_{l'} \omega_l + g_{l'}$*). Hence one can assume that both* $t_l \neq t_{l'}$ *and* $t_l \varpi_l + g_l \neq t_{l'} \varpi_l + g_{l'}$ *hold. Define now* $\varpi$ *by:*

$$\begin{aligned} t_l \varpi + g_{l'} &= t_l \varpi_l + g_l \\ &\Leftrightarrow \\ \varpi &= \varpi_l + \frac{g_l - g_{l'}}{t_l} \end{aligned}$$

*Clearly one has:*

$$\begin{aligned} U(t_l \varpi + g_{l'}, (1 - c - t_l)\omega_m) &= \Phi(t_l, \varpi, \omega_m, g_{l'}) \\ &= U(t_l \omega_l + g_l, (1 - c - t_l)\omega_m) \\ &= \Phi(t_l, \varpi_l, \omega_m, g_l, c) \end{aligned} \tag{26}$$

*for every household* $m$*. For this non-stratified jurisdiction structure to be stable, one must have:*

$$\begin{aligned} \Phi(t_l, \varpi_l, \omega_h, g_l, c) &\geq \Phi(t_{l'}, \varpi_{l'}, \omega_h, g_{l'}, c) \\ \Phi(t_l, \varpi_l, \omega_i, g_l, c) &\leq \Phi(t_{l'}, \varpi_{l'}, \omega_i, g_{l'}, c) \end{aligned}$$

*and*

$$\Phi(t_l, \varpi_l, \omega_k, g_l, c) \geq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_k, g_{l'}, c)$$

*or, using (26):*

$$\Phi(t_l, \varpi, \omega_h, g_{l'}, c) \geq \Phi(t_{l'}, \varpi_{l'}, \omega_h, g_{l'}, c) \tag{27}$$

$$\Phi(t_l, \varpi, \omega_i, g_{l'}, c) \leq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_i, g_{l'}, c) \tag{28}$$

*and*

$$\Phi(t_l, \varpi, \omega_k, g_{l'}, c) \geq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_k, g_{l'}, c) \tag{29}$$

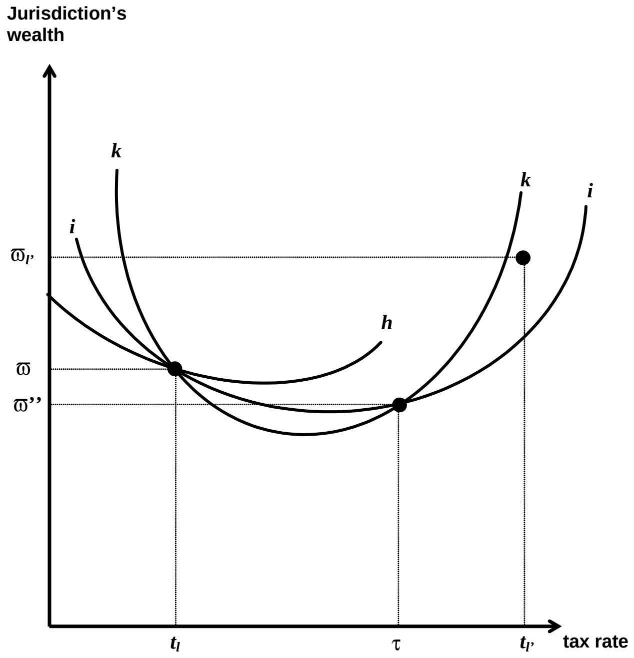


*Figure 5.*

*By lemma 4, the slopes of indifference curves in the space of all combinations of local tax rate and jurisdiction aggregate wealth are ordered by individual wealth for any level of central government grant and, therefore, for the level $g_{l'}$. Hence indifference curves of households h, i and k at $(t_l, \varpi)$ must be as they are depicted in figure 5. Clearly from this figure, unless indifference curves of households k and i or h and i cross in the "wrong" order at a point such as $(\tau, \varpi'')$, the set of combinations of local tax rates and jurisdiction aggregate wealth that i considers weakly worse (given central government grant $g_{l'}$) than $(t_l, \varpi)$ is contained in the set of such combinations that either household h or household k considers strictly worse than $(t_l, \varpi)$. Hence, unless indifference curves of two households cross in the wrong order at $(\tau, \varpi'')$ , inequalities (27)-(29) can not simultaneously hold for distinct combinations $(t_l, \varpi)$ and $(t_{l'}, \varpi_{l'})$ of local tax rates and jurisdiction tax rates. Hence the jurisdiction structure can not be stable.*

■

# 4 Conclusion

The main conclusion of this paper is that the welfarist intervention of a central government does not alter substantially the segregative properties of endogenous jurisdiction formation, at least when this jurisdiction formation is modelled within a framework *à la* Westhoff. Specifically, the GSC condition that it is necessary and sufficient to impose on household preferences for guaranteeing the wealth segregation of any stable jurisdiction is not affected by the presence of a central government as modelled in this paper. This of course does not mean that central government intervention does not affect jurisdiction formation. As the maxmin government case dramatically reveals, the redistributive behavior of the central government tends to reduce quite sharply the number of stable jurisdiction structures. Yet the stable jurisdictions structures that remain under a generalized utilitarian central government are segregated under exactly the same conditions on household's preferences than would be the case without a central government.

While we believe that the message according to which central government intervention does not modify the segregative forces underlying Tiebout-like processes of jurisdiction formation is of some interest, it is worth recalling the limitations of the analysis on which it stands. For one thing, the result is obtained, at least for its necessity part, under the assumption that preferences are additively separable. It would be nice to relax this assumption. Another limitation of the analysis lies, perhaps, in the simultaneous setting in which the decisions by households, central and local government are considered. As discussed in the paper, an alternative approach would be to assume a form of "leadership" of the central government in the process of jurisdiction formation. A third limitation of the analysis is the rather limited power given to the central government in our model to redistribute private wealth. Extending the analysis of this paper over these limitations, as well as many others, seems to us a worthy objective for future research.

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