The segregative properties of endogenous jurisdiction formation with a land market

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Abstract

This paper examines the segregative properties of Tiebout-like endogenous processes of jurisdiction formation in presence of a competitive land market. In the model considered, a continuum of households with different wealth levels and the same preferences for local public goods, private spending and housing choose a location from a finite set. Each location has an initial endowment of housing that is priced competively and that belongs to absentee landlords. Each jurisdiction is also endowed with a specific technology for producing public goods. Households' preferences are assumed to be homothetically separable between local public goods on the one hand and private spending and housing on the other. Public goods provision is financed by a given, but unspecified, mixture of (linear) wealth and housing taxes. We show that stable jurisdiction structures are always segregated by wealth only if households view any public good conditionally on the quantities of the other public goods as either always a gross substitute, or either always a gross complement, to private spending. We also show that, if there are more than one public good, this condition is not sufficient for segregation unless households preferences are additively separable. Since this condition is necessary and sufficient for the segregation of stable jurisdiction structures without land market and with only one public good, our results suggests that introducing a land market does not affect the segregative properties of endogenous jurisdiction formation but that increasing the number of public goods mitigates segregation.

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1 Introduction

It is widely believed that endogenous processes of jurisdiction formation, at least when driven by perfectly mobile households who make a [1] trade-off between local taxes and local public provision, are *self-sorting* and *segregative*. That is to say, such processes lead to the formation of homogenous jurisdictions inhabited by households with "similar" characteristics. [**?**] investigates the validity of this belief in the context of a classical model of endogenous jurisdiction formation due to [2]. In this model, households with different wealth and the same preference for local public spending and private spending choose to locate in a finite set of possible places of residence. Households who choose the same place form a jurisdiction that democratically decides of local taxes (paid by households on their private wealth) and local public spending. The analysis focuses on *stable* jurisdiction structures. These are partitions of the set of households that is immune to individual deviations, under the assumption that an individual move has no effect on jurisdictions' wealth, tax rates and public spending. [**?**] identify a condition on households' preference - the GSC condition - that is necessary and sufficient to ensure the wealth-segregation of any such stable jurisdiction structure. As in [2], the definition of "segregation" used by [**?**] is that underlying the notion of *consecutiveness* (see also [3]). It defines as "wealth-segregated" any jurisdiction structure in which the richest household of a jurisdiction is, weakly, poorer than the poorest household of any other jurisdiction with a strictly larger *per capita* wealth. The GSC condition requires households to consider local public spending to be either always a gross complement, or always a gross substitute, to the private good. In [4], the necessity and sufficiency of the GSC condition for segregation is also established for a generalized version of the model that allows for the presence of a generalized-utilitarian redistributive central government, under the additional assumption that households preferences are additively separable. While the GSC condition is stringent, and may be violated even by additively separable preferences, it is certainly not an outlandish condition. For this reason, [**?**] and [4] results may be seen as providing support to the widespread intuition that endogenous processes of jurisdiction formation *à la* Tiebout are inherently segregative.

Yet these results are obtained in a model with only one public good and without a dwelling market and where, as a result, households can reside for free in the jurisdiction that offers their favorite public good and tax package. This contrasts somewhat with *actual* processes of jurisdiction formation in which households must consume housing in order to have access to the public good and tax package available at a particular jurisdiction. Does the requirement for households to consume housing in order to benefit from local tax and public good package affect the segregative properties of endogenous processes of jurisdiction formation ? More specifically, what are the conditions - if any - that households preferences must satisfy in order for any stable jurisdiction structure to be segregative when housing consumption is required for living in a jurisdiction ? This is the main question examined in this paper. Realism is only one reason for addressing that question. Another motivation for identifying the conditions on households preferences that are necessary and sufficient for the segregation of stable jurisdiction structure in presence of housing markets is that those markets provide an easy way of testing the conditions through hedonic methods (see e.g. [5] or [6]).

Addressing this question requires of course a model of jurisdiction formation driven by household's optimizing decisions with respect to local public good and taxes under perfect mobility that incorporates housing consumption. At least three approaches to this modeling have been proposed in the literature.

The first, explored notably by [**?**], [7], [8], assumes that housing, owned by absentee landlords, is a perfectly divisible good available in (possibly) different quantities in a finite number of locations. Households have preference for housing, local public spending and non-housing private spending and must consume a positive amount of housing at most one place. Purchase of housing is made on competitive markets that equalize local supply of housing with local demand. Households who choose to consume housing at the same location form a jurisdiction and choose by majority voting a *property tax* rate which, when applied to the (before tax) market value of local land, finances local public spending. An important issue discussed in this literature is the difficulty of establishing *existence* of stable jurisdiction structures. [7] and [8] have provided conditions on households' preferences that are sufficient for the existence of stable jurisdiction structures. As shown by these authors, the conditions also guarantee that the stable jurisdiction structures will be "segregated" in the same (consecutive) sense than above. Models that satisfy these assumptions have been the object of intensive empirical research in the last ten years or so (see e.g. [9], [**?**], [**?**] and [**?**]).

Another approach, explored by [10], [11] and [12] among others, considers a similar setting than the previous stream but with the important difference that local public good provision is assumed to be financed by wealth taxation. To that extent, this setting is closer in spirit to that of Westhoff to which it is easy to compare. Another advantage of this setting is that it eases considerably the problem of existence of stable jurisdiction structures (see for instance [12] for a quite general existence theorem). On the other hand, assuming the financing of local public spending by wealth taxation is clearly at odd with what is observed in most institutional settings that we are aware, as property tax is by far the most widely use financing device for local public spending.

The third approach has been proposed by [13], and builds on the (largely unpublished) work of [14], [15] and [16]. It differs from the two previous ones in that it assumes that housing is an indivisible good that is available in various types in the various jurisdictions. In such a setting, [13] proves the existence of stable jurisdiction structure under both wealth and dwelling taxation. He also provides conditions that are sufficient for the segregation of stable jurisdiction structures.

In this paper, we stick to the perfectly divisible land (or housing) framework but we consider a financing scheme of local public good provision that combines wealth and dwelling taxation. Moreover, we allow for the possibility that jurisdictions produce several public goods rather than a single one. Yet, we adopt a more general view than the one typically taken in the literature with respect to the mechanism used by local jurisdictions to select public good provisions and taxes. Indeed, except for the linearity of the taxation (on both housing and wealth) and the balancing of the local government budget, we do not make any assumption on the process by which taxes and local public good are chosen. By contrast, much of the literature assume that local taxes are chosen by some voting mechanism (e.g. tax rates are the favorite ones of the median individual). It is actually this voting assumption which, together with competitive pricing of lands, create problems of existence of stable jurisdiction structures. By abstracting from the particular mechanism used by jurisdictions to decide upon local public good provision and taxes, our analysis thus escapes from the difficulties raised by possible inexistence of stable jurisdiction structures.

While the abstraction from the particular intra-jurisdiction collective choice goes toward a generalization of the approach favoured in the literature, we conduct the analysis by making the (significantly) simplifying assumption that households preferences are *homotheticaly separable* in the sense of [17] (3.4.2) between local public goods on the one hand and private spending (on housing and private consumption) on the other. This assumption is admittedly restrictive. Yet, it is not grossly inconsistent with the available empirical evidence (see for instance [**?**]) that indicates that the budget share devoted to housing is remarkably stable both across locations (who differ in local public good provision) and across households (who differ in their wealth).

In this framework, we show that the propensity of stable jurisdiction structures to lead to segregation is not affected by the introduction of the housing market. For we show that a suitable generalization of the GSC condition remains necessary and sufficient for any stable jurisdiction to be segregated if there is only one public good. While we interpret this result as indicative of a somewhat robust connection between the GSC condition and the segregative properties of endogenous jurisdiction formation, we emphasize that the assumption of separable homotheticity is crucial for the result. Moreover, our analysis also suggests that the GSC condition may not be sufficient for segregation if preferences are not additively separable and there are more than one public goods. This suggests, somewhat intuitively, that increasing the number of public goods by which jurisdictions can be distinguished may mitigate the segregative feature of endogenous jurisdiction formation.

The plan of the rest of this paper is as follows. In the next section, we introduce the main notation and concepts. Section 3 provide the results for housing taxation while section 4 show how the results extend to the case with a welfarist central government. Section 5 provides some conclusion.

2 The model

As in the literature, we consider economies with a continuum of households represented by the $[0,1]$ interval, and we denote by λ the Lebesgue measure defined over all (Lebesgue measurable) subsets of $[0,1]$. For any Lebesgue measurable subset S of $[0,1]$, we interpret $\lambda(S)$ as "the fraction of households" in the set S. An economy is made of the three following ingredients.

First, there is a *wealth distribution* modeled as a Lebesgue measurable, increasing and bounded from above function $\omega : [0,1] \to \mathbb{R}_{++}$ that associates to every household $i \in [0,1]$ its strictly positive private wealth ω_i. Limiting attention to increasing functions is a convention according to which households are ordered by their wealth ($i \leq i' \Longrightarrow \omega_i \leq \omega_{i'}$).

The *second* ingredient in the description of an economy is a specification of the households' preferences, taken to be the same for all households. These preferences are defined over k local public goods ($\mathbf{Z} = (Z_1, ..., Z_k)$), private spending ($x$) and housing ($h$) and are represented by a twice differentiable, strictly increasing and strictly quasi-concave[1] utility function $U : \mathbb{R}_+^{k+2} \to \mathbb{R}$. We sometimes focus attention on some particular public good j. On such occasions, we may find convenient to write a particular bundle $\mathbf{Z}$ of public goods as $\mathbf{Z} = (Z_j; \mathbf{Z}_{-j})$ where the bundle $\mathbf{Z}_{-j}$ of the $k-1$ other public goods is defined by $Z_{-jh} = Z_h$ if $h < j$ and $Z_{-jh} = Z_{h+1}$ if $h > 1$. All preferences that are considered in this paper are also assumed to satisfy the following *regularity condition* with respect to the public goods.

Condition 1 *(Regularity) If $k \geq 2$, then, for any bundle $(x,h) \in \mathbb{R}_+^2$ of private goods and any two bundles $\mathbf{Z}$ and $\mathbf{Z}' \in \mathbb{R}_+^k$ of public goods, there exists a public good $j \in \{1, ..., k\}$ for which a quantity quantity $\overline{Z}_j \in \mathbb{R}_+$ can be found for which $U(\overline{Z}_j; \mathbf{Z}_{-j}, x, h) = U(\mathbf{Z}', x, h)$.*

This weak regularity condition, that applies only if there are more than one public good, rules out the possibility for indifference surfaces in the public goods space (conditional on any bundle of private goods) to have "vertical or horizontal" asymptotes in the interior of $\mathbb{R}_+^k$. If indifference surfaces in

[1] A function $f : A \to \mathbb{R}$ (where A is some convex subset of $\mathbb{R}^l$) is strictly quasi-concave if, for every a, b, $x \in A$, with $a \neq b$ and every $\alpha \in]0,1[$, $f(a) \geq f(x)$ and $f(b) \geq f(x)$ imply $f(\alpha a + (1-\alpha)b) > f(x)$.

the public goods space have vertical or horizontal asymptotes, then these asymptotes must be the axes of the plane. For instance, preference that would generate indifference curves in $\mathbb{R}_+^2$ as in figure 1 below, are ruled out by this condition.

itbpFUX3.5829in2.3886in0inFigure 1Plot

As mentioned above, we also assume that the households preferences are *homothetically separable*, in the sense of [17] (3.4.2) between the k public goods on the one hand and the two private goods on the other. This assumption amounts to say that, for all $\mathbf{Z} \in \mathbb{R}_+^k$ and $(x, h) \in \mathbb{R}_+^2$, U can be written as:

$$U(\mathbf{Z}, x, h) = G(\mathbf{Z}, \Phi(x, h)) \tag{1}$$

for some twice continuously differentiable increasing and strictly quasi-concave function $G : \mathbb{R}_+^{k+1} \to \mathbb{R}$ and some twice continuously differentiable, increasing and homogenous of degree 1 function $\Phi : \mathbb{R}_+^2 \to \mathbb{R}_+$. For proving the sufficiency of the GSC condition when the number of public goods is larger than 1, we shall assume that households preference are not only homothetically separable but are also *additively separable* so that the function G of expression (**??**) can be written, for any $Z \in \mathbb{R}_+^k$ and $\phi \in \mathbb{R}_+$, as $G(Z, \phi) = g(Z) + \Gamma(\phi)$ for some increasing and strictly quasi-concave function $g : \mathbb{R}_+^k \to \mathbb{R}_+$. and some continuous and increasing function $\mathbb{R}_+ \to \mathbb{R}$.

We denote by $Z_j^M(\mathbf{p}^Z; p_x, p_h, R)$, $x^M(\mathbf{p}^Z; p_x, p_h, R)$ and $h^M(\mathbf{p}^Z, p_x, p_h, R)$ the household's Marshallian demands for public good j (for $j = 1, ..., k$), private consumption and housing (respectively) when the prices of local public goods are $\mathbf{p}^Z = (p_1^Z, ..., p_k^Z)$, the prices of private spending and housing are p_x and p_h and when its income is R. These Marshallian demand functions are the - unique under our assumptions - solution of the program:

$$\max_{Z,x,h} U(\mathbf{Z}, x, h) \text{ subject to } \mathbf{p}^Z.\mathbf{Z} + p_x x + p_h h \leq R \tag{2}$$

and are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}_+^{k+2}$).

We emphasize that we view Marshallian demand functions as a dual way of representing households preference for the $k + 2$ goods rather than as a behavioral description of households behavior. After all real households rarely purchase local public goods on competitive markets. For any given vector $\mathbf{Z} \in \mathbb{R}_+^k$ of public goods, we denote by $V^{\mathbf{Z}}$ the conditional (upon $\mathbf{Z}$) indirect utility function defined, for any $(p_x, p_h, y) \in \mathbb{R}_+^3$, by:

$$V^{\mathbf{Z}}(p_x, p_h, y) = \max_{x,h} U(\mathbf{Z}, x, h) \text{ subject to } p_x x + p_h h \leq y \tag{3}$$

This conditional indirect utility function plays an important role in the analysis. It describes indeed the maximal utility achieved by a household endowed with a (net of tax) wealth y/p_x (using private good as *numéraire*) when living in a jurisdiction offering quantities Z of the local public goods and (net of tax) dwelling price p_h/p_x. We denote by $h^{\mathbb{Z}M}$ and $x^{\mathbb{Z}M}$ the conditional (upon $\mathbf{Z}$) Marshallian demands of the two private goods that solve program 3. Under our assumptions, these conditional Marshallian demands are continuous functions of their three arguments that satisfy all the usual properties of Marshallian demands.

We also note that, thanks to homothetic separability, it is possible to describe program (2) by means of a *two-step budgeting* procedure (see e.g. [17], ch. 5).

The *first* step of the procedure is described by the program:

$$\max_{(Z;\phi)\in\mathbb{R}_+^{k+1}} G(\mathbf{Z},\phi) \text{ s. t. } \mathbf{p}^Z.\mathbf{Z}+\overline{E}(p_x,p_h)\phi \le R. \tag{4}$$

where the function $\overline{E}:\mathbb{R}_+^2$ is defined by the dual program:

$$\overline{E}(p_x,p_h)\phi = E(p_x,p_h,\phi)=\min_{x,h} p_x x + p_h h \text{ s. t. } \Phi(x,h)\ge\phi \tag{5}$$

As is well-known indeed the expenditure function associated to a homogeneous utility function is linear in utility. As is also well-known from standard microeconomic theory that $\overline{E}$ is continuous, homogeneous of degree 1, increasing and concave. Hence, in this first step, the household is depicted as allocating its wealth R between the local public goods $\mathbf{Z}$ and the "utility of private spending" (measured by $\phi=\Phi(x,h)$), taking as given the public price vector p^Z and the "aggregate" price $p^X=\overline{E}(p_x,p_h)>0$ of (the utility of) private spending. We notice that the function $\overline{E}$ is also involved in the definition of the conditional indirect utility function $V^{\mathbf{Z}}$ defined in program (3). Indeed, it is immediate to verify that $V^{\mathbf{Z}}$ writes:

$$V^{\mathbf{Z}}(p_x,p_h,R)=G(\mathbf{Z},\frac{R}{\overline{E}(p_x,p_h)}) \tag{6}$$

Denote now by $\mathbf{Z}^*(\mathbf{p}^Z,\overline{E}(p_x,p_h),R)$ and $\phi^*(\mathbf{p}^Z,\overline{E}(p_x,p_h),R)$ the (unique under our assumptions) solution to program (4). Denote also by $e(\mathbf{p}^Z,p_x,p_{h,},R)$ the optimal expenditure on the "utility of private spending" that results from the solution of program (4) and that is defined by:

$$e(\mathbf{p}^Z,p_x,p_{h,},R)=\overline{E}(p_x,p_h)\phi^*(\mathbf{p}^Z,\overline{E}(p_x,p_h),R) \tag{7}$$

The *second* step of the procedure consists in solving the program :

$$\max_{x,h}\ \Phi(x,h) \text{ s. t. } p_x x+p_h h\le e(\mathbf{p}^Z,p_x,p_{h,},R)) \tag{8}$$

Denote by $x^*(p_x,p_h,e(\mathbf{p}^Z,p_x,p_{h,},R))$ and $h^*(p_x,p_h,e(\mathbf{p}^Z,p_x,p_{h,},R))$ the solution of program (8). Thanks to the homotheticity of the (separable from the public goods) preferences for private goods represented by the function

Φ, we know from standard microeconomic theory that the functions x^* and h^* so defined can be written as:

$$x^*(p_x, p_h, y) = \digamma^x(p_x, p_h)y$$

and

$$h^*(p_x, p_h, y) = \digamma^h(p_x, p_h)y$$

where, thanks to Roy's identity, $\digamma^x$ and $\digamma^h$ can be written as

$$\digamma^h(p_x, p_h) = \frac{\partial \overline{E}(p_x, p_h)/\partial p_h}{\overline{E}(p_x, p_h)} \tag{9}$$

and :

$$\digamma^x(p_x, p_h) = \frac{\partial \overline{E}(p_x, p_h)/\partial p_x}{\overline{E}(p_x, p_h)} \tag{10}$$

Moreover, one has

$$h^{\mathbf{Z}M}(p_x, p_h, y) = h^*(p_x, p_h, y)$$

and

$$x^{\mathbf{Z}M}(p_x, p_h, y) = h^*(p_x, p_h, y)$$

for any bundle $\mathbf{Z}$ of local public goods (Marshallian demands of housing and private spending are independent from public good provision). As a result of theorem 5.8 in [17], it follows that:

$$\begin{aligned} Z^M(\mathbf{p}^Z, p_x, p_h, R) &= Z^*(\mathbf{p}^Z; \overline{E}(p_x, p_h), R) & (11) \\ x^M(\mathbf{p}^Z, p_x, p_h, R) &= \digamma^x(p_x, p_h)e(\mathbf{p}^Z, p_x, p_{h,}, R) & (12) \\ h^M(\mathbf{p}^Z, p_x, p_h, R) &= \digamma^h(p_x, p_h)e(\mathbf{p}^Z, p_x, p_{h,}, R) & (13) \end{aligned}$$

These results will be used later on. We finally notice that this description of the household's preference is valid for any number of public goods whatsoever. As it turns out, a large part of the analysis deals with *conditional Marshallian demand* functions which are denoted, for any public good $j = 1, ..., k$, and any given specification $\overline{\mathbf{Z}}_{-j} = (\overline{Z}_1, ..., \overline{Z}_{j-1}, \overline{Z}_{j+1}, ..., \overline{Z}_k) \in \mathbb{R}_+^{k-1}$ of the quantities of the $k-1$ other public goods, by $Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z; p_x, p_h, R)$, $x^M(p_j^Z; p_x, p_h, R)$ and $h^M(\overline{\mathbf{Z}}_{-j}; p_j^Z; p_x, p_h, R)$. These conditional demands are defined to be the (unique) solution of program (2) for the conditional utility function $U^{\overline{\mathbf{Z}}_{-j}} : \mathbb{R}_+^3 \to \mathbb{R}$ defined, for any $(Z_j,h,x) \in \mathbb{R}_+^3$, by:

$$U^{\overline{\mathbf{Z}}_{-j}}(Z_j, x, h) = U(\overline{Z}_1, .., \overline{Z}_{j-1}, Z_j, .\overline{Z}_{j-1}, ..., \overline{Z}_k, x, h) \tag{14}$$

It can be checked that the aforementioned properties of U, including homothetic separability, are also possessed by $U^{\overline{\mathbf{Z}}_{-j}}$ for any $\overline{\mathbf{Z}}_{-j} \in \mathbb{R}_+^{k-1}$. We accordingly denote by $G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}$ the (conditional) specification of the function G of expression (**??**) above when U is replaced by $U^{\overline{\mathbf{Z}}_{-j}}$.

The *last two* elements of our description of an economy are a *common finite* set $\mathbb{L}$ of $\bar{l}$ locations available to households together with a specification, for each location $l \in \mathbb{L}$, of the amount of land $L^l \in \mathbb{R}_{++}$ exogenously assigned to l, and, for each public good $j = 1, ..., k$, of a cost function $C_j^l : \mathbb{R}_+ \to \mathbb{R}_+$ of producing public good j at l. We assume that these cost functions satisfy $C_j^l(0) = 0$ and are increasing at every location l and for every public good j. Allowing the cost of producing a given public good to differ across locations seems natural to us (it is more costly to provide a given access to a sand beach in Paris than in Miami). We assume throughout that the endowments of land belong to absentee landlords that play no role in the economy. We denote by $\mathbb{D}$ the domain of all economies $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ that satisfy these assumptions and by $\mathbb{D}^A$ the subset of $\mathbb{D}$ that made of economies where households have additively separable preferences.

A **jurisdiction structure** for the economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ is a list $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l\in\mathbb{L}})$ where :

- $j : [0,1] \rightarrow_{l\in\mathbb{L}} \{l\} \times \mathbb{R}_{++}$ is a Lebesgue measurable function that assigns to each household i in $[0,1]$ a unique combination $j(i) = (l_i, h_i^l)$ of a place of residence and a housing consumption at that place of residence and, for every $l \in \mathbb{L}$:
- $p^l \in \mathbb{R}_{++}$ is the (before tax) housing price at location l.
- $t_h^l \in \mathbb{R}_{++}$ is the housing tax rate prevailing at location l
- $t_w^l \in [0,1]$ is the wealth tax rate prevailing at location l
- $\mathbf{Z}^l \in \mathbb{R}_+^k$ is the bundle of local public goods available at location l.

For any such jurisdiction structure, and for any $l \in \mathbb{L}$, we denote by $j^l = \{i \in [0,1] : j(i) = (l, a) \text{ for some } a > 0\}$. Hence j^l is the (Lebesgue measurable) set of all households who have chosen to locate at l in the considered jurisdiction structure. The possibility that $\lambda(j^l) = 0$ (l is a "desert" jurisdiction) is of course not ruled out. We restrict attention throughout to **feasible** jurisdiction structures that satisfy the additional conditions that:

$$\int_{j^l} h_i^{l_i} d\lambda \leq L^l \text{ and} \tag{15}$$

$$t_h^l p^l \int_{j^l} h_i^{l_i} d\lambda + t_w^l \int_{j^l} \omega_i d\lambda \geq {}_{h=1}^{k} C_h^l(Z_h^l) \tag{16}$$

at every location l. That is to say, feasible jurisdiction structures are such that, at any location, aggregate housing consumption does not exceed the total amount of land that is available there (15), and that tax revenues raised are sufficient to cover the cost of providing the available bundle of public goods (16). Notice that the later inequality, together with the assumption that the cost functions are increasing and satisfy $C_j^l(0) = 0$, implies that the only public good package Z^l that can be observed in a "desert" jurisdiction is $Z^l = 0^l$.

Given an economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l \in \mathbb{L}})$ and a jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l \in \mathbb{L}})$ for this economy, we denote by $H^l = \int_{j^l} h_i^l d\lambda$ and $\varpi^l = \int_{j^l} \omega_i d\lambda$ the aggregate consumption of land and wealth (respectively) at location l.

We remark that our definition of jurisdiction structures is quite general and covers several models of endogenous jurisdiction formation with a housing market examined in the literature. It covers in particular models like [12] where local public good provision is assumed to be financed by (linear) wealth taxation as well as models such as [**?**], [7], [8], [18], [9], [**?**], [**?**], [**?**]) in which local public goods are financed by dwelling (or property) tax only. We believe that allowing for both types of taxation is consistent with what is observed in several real world institutional settings. For instance several states in the US have "property tax relief" features that reduce the tax burden of specific categories of tax payers on the basis of their income. Similarly in France, many households are exempt from the so-called "taxe d'habitation" (dwelling tax) on the basis of their income.

We now turn to our definition of a *stable jurisdiction structure*, which we formally state as follows.

Definition 1 *A feasible jurisdiction structure* $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l \in \mathbb{L}})$ *for the economy* $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l \in \mathbb{L}})$ *is stable if, for every* $l \in \mathbb{L}$ *such that* $\lambda(j^l) > 0$, *one has, for every* $i \in j^l$, $H^l = L^l$ *and* $U(\mathbf{Z}^l, \omega_i(1 - t_w^l) - p^l(1 + t_h^w)h_i^l, h_i^l) \geq V^{\mathbf{Z}^{l'}}(1, p^{l'}(1 + t_h^{l'}), \omega_i(1 - t_w^{l'}))$ *for every* $l' \in \mathbb{L}$ *not necessarily distinct from* l.

In words, a feasible jurisdiction structure is stable if, in every jurisdiction, land consumption is equal to land supply and the bundle of land and private spending obtained by every household is considered, by this household, better than any bundle that it could afford (given its wealth, dwelling tax and land prices) either in its jurisdiction of residence or elsewhere.

We notice that our definition of stability does not assume any specific mechanism for choosing local public good provision and tax rate. By contrast, much of the literature dealing with endogenous formation of jurisdictions assumes that local jurisdiction choose their tax rate and/or public good provision by some voting mechanism. It is well-known that combining a voting mechanism with a competitive pricing of the land may raise serious existence problem, that are exacerbated if voting concerns the housing tax rate (see e.g. [**?**], [7], [8], [18] and [9]). These existence problems are alleviated here by avoiding the requirement for the tax rate to be result of a voting procedure. For instance the (grand) jurisdiction structure in which all households are put into one jurisdiction will be stable if a Inada's condition on one of the public good is assumed (as no household would want to unilaterally move to a desert jurisdiction with zero public goods in that case, even if land is free).

We now investigate under which condition any stable jurisdiction structure is wealth-segregated. This requires a definition of wealth-segregation that we provide as follows.

Definition 2 *A feasible jurisdiction structure* $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l \in \mathbb{L}})$ *for the economy* $(\omega, U, \mathbb{L}, \{L_l, C_1^l, ..., C_k^l\}_{l \in \mathbb{L}})$ *is wealth-segregated if, for every households* h*,* i *and* $k \in [0,1]$ *such that* $\omega_h < \omega_i < \omega_k$*,* $h \in j^l$*,* $k \in j^l$ *and* $i \in j^{l'}$ *for some* $l' \neq l$ *imply that* $V^{\mathbf{Z}^l}(1, p^l(1+t_h^l), \omega_h(1-t_w^l)) = V^{\mathbf{Z}^{l'}}(1, p^{l'}(1+t_h^{l'}), \omega_h(1-t_w^{l'}))$ *for every* $h \in j^l \cup j^{l'}$*.*

In words, a jurisdiction structure is wealth-segregated if any jurisdiction j^l containing two households with strictly different levels of wealth also contains any household with wealth in between of those two or, if it does not contain this household, it is because it resides in another jurisdiction $j^{l'}$ that is "identical" to jurisdiction j^l in the sense that everybody living in either of these jurisdiction is indifferent between the two.

In [**?**], in a model without housing and with one public good, it was established that the Gross Substitutability/Complementarity (GSC) condition according to which the (non-symmetric) relation of gross substitutability or complementarity (as it may be) of the unique public good *vis-à-vis* private consumption is independent from all prices is necessary and sufficient for the wealth segregation of any stable jurisdiction structure. In the current context where land is present and where there are, possibly, many public goods, it turns out that the GSC condition applied to the conditional Marshallian demand of every local public is necessary and sufficient for securing the wealth-segregation - as per definition 2 - of any stable jurisdiction structure as per definition 1. This statement of this generalized GSC condition is the following.

Condition 2 *(Generalized GSC) The household's preference satisfies the generalized GSC condition if, for every local public good* j*, the function* $Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x, p_h, R)$ *is monotonic with respect to* p_x *for any* $(p_j^Z, p_h, R) \in \mathbb{R}_+^3$*.*

We notice that, thanks to (11), one can write

$$Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x, p_h, R) = Z^{\overline{\mathbf{Z}}_{-j}*}(p_j^Z; \overline{E}^{\overline{\mathbf{Z}}_{-j}*}(p_x, p_h), R) \tag{17}$$

where the functions $Z^{\overline{\mathbf{Z}}_{-j}*}$ and $\overline{E}^{\overline{\mathbf{Z}}_{-j}*}$ are nothing else than the functions Z^* and $\overline{E}$ defined for the utility function $U^{\overline{\mathbf{Z}}_{-j}}$. Since the function $\overline{E}^{\overline{\mathbf{Z}}_{-j}*}$ is increasing in its two arguments, requiring the function $Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x, p_h, R)$ to be monotonic with respect to p_x is equivalent to requiring the function $Z^{\overline{\mathbf{Z}}_{-j}*}$ to be monotonic with respect to the *aggregate* private goods price index $p_X^{\overline{\mathbf{Z}}_{-j}*} = \overline{E}^{\overline{\mathbf{Z}}_{-j}*}(p_x, p_h)$.

Using this fact, we now establish that this generalization of the GSC condition is necessary and sufficient for the wealth segregation of any stable jurisdiction structure. As it turns out, this condition will not be sufficient in the most general version of the model presented here. It will be sufficient either if we make the additional assumption that there is only one local public good, or that the household's preferences are additively separable between local public goods on the one hand and the private goods on the other.

Yet, as established in the following proposition, the condition will be necessary for segregation.

Proposition 1 *The GSC condition is necessary for the wealth segregation of any stable jurisdiction structure for an economy* $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l \in \mathbb{L}})$ *in* $\mathbb{D}$.

Proof. Suppose that the GSC condition is violated. This means that there exists a public good $j \in \{1, ..., k\}$, some private spending prices p_x^0, p_x^1 and p_x^2 satisfying $0 < p_x^0 < p_x^1 < p_x^2$ for which one has:

$$Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^0, p_h, R) = Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^2, p_h, R) > Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^1, p_h, R) \quad (18)$$

(the argument is similar if we assume instead $Z_j^{MC}(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^0, p_h, R) = Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^2, p_h, R) < Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^1, p_h, R)$) for some public good j price $p_j^Z \in \mathbb{R}_{++}$, housing price $p_h \in \mathbb{R}_{++}$, some vector $\overline{\mathbf{Z}}_{-j} \in \mathbb{R}_+^{k-1}$of quantities of the other public goods and some income $R > 0$). Denote by $\mathbf{Z}^1$ and $\mathbf{Z}^2$ the vector of public goods defined by:

$$\mathbf{Z}^1 = (Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^1, p_h, R); \overline{\mathbf{Z}}_{-j})$$

and:

$$\begin{aligned} \mathbf{Z}^2 &= (Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^2, p_h, R); \overline{\mathbf{Z}}_{-j}) \\ &= (Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^0, p_h, R); \overline{\mathbf{Z}}_{-j}). \end{aligned}$$

Using (17)) and homogeneity of degree 0 of Marshallian demands, expression (18) can be written as:

$$\begin{aligned} Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^0, p_h), 1) &= Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1) \quad (19) \\ &> Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1) \end{aligned}$$

where:

$$\begin{aligned} q_j^Z &= p_j^Z/R \text{ and} \\ \widetilde{E}(p_x^j, p_h) &= \overline{E}^{\overline{\mathbf{Z}}_{-j}*}(p_x^j, p_h)/R \text{ for } j = 0, 1, 2 \end{aligned}$$

Of course, since $0 < p_x^0 < p_x^1 < p_x^2$ and the function $\overline{E}^{\overline{\mathbf{Z}}_{-j}*}$ is increasing with respect to its arguments, one has $0 < \widetilde{E}(p_x^0, p_h) < \widetilde{E}(p_x^1, p_h) < \widetilde{E}(p_x^1, p_h)$. Let us show that we can find an economy in $\mathbb{D}$ for which a stable jurisdiction structure can be non-segregated. For this sake, consider the economy where $\mathbb{L} = \{1, 2\}$ and where the wealth distribution function ω is such that there are α and $\beta \in]0, 1[$ satisfying $\alpha < \beta$ for which one has:
$\omega_i = 1/\widetilde{E}(p_x^0, p_h)$ for all $i \in [0, \alpha[$,
$\omega_i = 1/\widetilde{E}(p_x^1, p_h)/R$ for all $i \in [\alpha, \beta[$ and,
$\omega_i = 1/\widetilde{E}(p_x^2, p_h)/R$ for all $i \in [\beta, 1]$,
and let there be a mass $\mu_t > 0$ of household of type t (for $t = 0, 1, 2$) with the masses chosen in such a way as to satisfy:

$$\mu_0\omega_0 + \mu_2\omega_2 = \frac{1}{q_j^Z} = \mu_1\omega_1 \tag{20}$$

It is clearly possible to find such positive real numbers μ_t (for $t = 0, 1, 2$). One simply set

$$\mu_1 = \frac{1}{\omega_1 q_j^Z} > 0$$

and observe that there are several strictly positive values of μ_0 and μ_2 that satisfy:

$$\mu_2 = \frac{1}{\omega_2}(\frac{1}{q_j^Z} - \omega_0\mu_0)$$

Consider any increasing and convex cost functions C_g^l (for $g = 1, ..., k$) and $l = 1, 2$) such that ${}_{g\neq j}C_g^1(Z_g) = {}_{g\neq j}C_g^1(Z_g) = c$ for some non-negative real number c that we leave, for the moment, unspecified. Let also $C_j^1 = C_j^2 = C$ for some **strictly increasing** and convex cost function C satisfying:

$$\begin{aligned} C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)) &< \\ C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^0, p_h), 1)) &= C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1)) \\ &< \mu_0\omega_0 + \mu_2\omega_2 \\ &= \mu_1\omega_1 \end{aligned} \tag{21}$$

Consider the jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l=1,2})$ defined by:

$$j(i) = (2, F^h(p_x^0, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^0, p_{h,}, 1)) \text{ for } i \in [0, \alpha[, \tag{22}$$

$$j(i) = (1, F^h(p_x^1, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1)) \text{ for } i \in [\alpha, \beta[, \tag{23}$$

$$j(i) = (2, F^h(p_x^2, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^2, p_{h,}, 1)) \text{ for } i \in [\beta, 1], \tag{24}$$

$$p^1(1+t_h^1) = p^2(1+t_h^2) = p_h \tag{25}$$

$$t_h^1 = t_h^2 = 0,$$

$$t_w^2 = q_j^Z C(Z_j^{\overline{Z}-j*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1)) \text{ and} \tag{26}$$

and

$$t_w^1 = q_j^Z C(Z_j^{\overline{Z}-j*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)) \tag{27}$$

where the function $e^{\overline{Z}-j}$ is the analogue, for program 14, to the function e defined in expression (7) for program (4). Observe that, thanks to (20) and (21), one has $t_w^l \in [0,1]$ for $l = 1,2$. Observe also that equation (25) leaves complete freedom for choosing before-tax housing price and dwelling tax rates p^l and t_h^l satisfying $p^l(1+t_h^l) = p_h$ for $l = 1,2$. Set now the quantities of land $L^1$and L^2 so that:.

$$\mu_1 F^h(p_x^1, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1) = L^1 \tag{28}$$

and:

$$\mu_0 F^h(p_x^2, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^2, p_{h,}, 1) + \mu_2 F^h(p_x^2, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^2, p_{h,}, 1) = L^2 \tag{29}$$

There are clearly no difficulties in finding such L^1 and L^2. Given L^l, set the yet undetermined positive real numbers t_h^l and p^l so that :

$$t_h^l p_h^l L^l = c$$

(for $l = 1,2$). It is clear that this equality, which says that the cost of producing the public goods other than j in the two jurisdiction is financed by housing taxation, requires to set $t_h^l = \frac{c}{p_h^l L^l}$ for $l = 1,2$. Substituting this back into equation (25) yields:

$$p^l = p_h - \frac{c}{L^l}$$

for $l = 1,2$. It is clear that the before tax housing price p^l so defined will be positive if the cost functions C_g^l (for $g = 1,...,k$) and $l = 1,2$) are chosen in such a way that c is sufficiently small. Let us show that this non-segregated jurisdiction structure is stable. Our choice of L^1 and L^2 already guarantees (equations (28)-29) that land markets clear at the two locations. Since, as was just established, the cost of producing public goods other than j in the two jurisdiction is exactly covered by housing tax revenues, equations (26)-(27) imply that the total tax raised in each of the two jurisdictions covers exactly the cost of public good provision. We only need to show that no household has incentive to modify its consumption of public good and private goods either at its location or at the other. We provide the argument for a household of type 1 living at jurisdiction 1, leaving to the reader the task of verifying that the same argument holds for a type 0 and type 2 household living at jurisdiction 2 Consider therefore household 1 who has private $\omega_1 = 1/\widetilde{E}(p_x^t, p_h)$ and who lives at location 1 where it consumes the bundle $\mathbf{Z}^1 = (Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)), \overline{Z}_{-j})$ of public goods and has $\omega_1(1 - t_w^1)$ to spend on housing and private spending. Yet:

$$\begin{aligned}
\omega_1(1 - t_w^1) &= \omega_1[1 - q_j^Z C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1))] \\
&= \frac{[1 - q_j^Z C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1))]}{\widetilde{E}(p_x^1, p_h)} \\
&= \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1) \qquad (30)
\end{aligned}$$

thanks to the budget constraint associated to the program 4 applied to the conditional consumer's program (14). Now, since a type 1 household consumes $\digamma^h(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1)$ units of housing (equation (22) purchased at price p_h, such a household has

$$\omega_1(1 - t_w^1) - p_h \digamma^h(p_x^1, p_h)[e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1)]$$

available for private spending. Yet we know that

$$\omega_t(1 - t_w^1) - p_h \digamma^h(p_x^t, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1)$$

$$\begin{aligned}
&= \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1) - p_h \digamma^h(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1) \\
&= p_x^t \digamma^x(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1)
\end{aligned}$$

Since $(\digamma^h(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1), \digamma^x(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1))$ solves program (8), a type 1 - household can not find a bundle of private spending x and housing h satisfying $p_h h + x \leq \phi^{\overline{Z}_{-j}*}(q_j^Z, \overline{E}(p_x^1, p_h), 1)) = \omega_t(1 - t_w^1)$ that is strictly preferred to:

$$(\digamma^h(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1), p_x^1 \digamma^x(p_x^1, p_h)[e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1)]$$

Hence a type 1 household has no incentive to change its private consumption pattern within its jurisdiction. It has also no incentive to move to location 2. Indeed, if it were to move there, it would obtain the bundle:

$$\mathbf{Z}^2 = (Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^0, p_h), 1)), \overline{Z}_{-j}) = (Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1)), \overline{Z}_{-j})$$

for which it would pay $t_w^2 \omega_1$ amount of tax and would have $\omega_1(1 - t_w^2)$ units of numéraire to spend on private matters. Observe now that, thanks to (26) and (30):

$$
\begin{aligned}
[t_w^1 + (1-t_w^1)]\omega_1 &= C(Z_j^{\overline{Z}_{-j}*}(q_j^Z;\widetilde{E}(p_x^1,p_h),1))/\mu_1 + \phi^{\overline{Z}_{-j}*}(q_j^Z,\widetilde{E}(p_x^1,p_h),1) \\
&= [t_w^2 + (1-t_w^2)]\omega_1 \\
&= C(Z_j^{\overline{Z}_{-j}*}(q_j^Z;\widetilde{E}(p_x^2,p_h),1))/\mu_1 + (1-t_w^2)]\omega_1
\end{aligned}
$$

Define now the function $\widetilde{G}^{\overline{Z}_{-j}}:\mathbb{R}_+^2 \to \mathbb{R}_+$. by:

$$\widetilde{G}^{\overline{Z}_{-j}}(c,\phi) = G^{\overline{Z}_{-j}}(C^{-1}(c),\phi)$$

where C^{-1} is the inverse cost function. This function is well-defined if C is strictly increasing. Since $(C(Z_j^{\overline{Z}_{-j}*}(q_j^Z;\widetilde{E}(p_x^1,p_h),1)),\phi^{\overline{Z}_{-j}*}(q_j^Z,\widetilde{E}(p_x^1,p_h),1))$ solves the program:

$$\max_{(Z;\phi)\in\mathbb{R}_+^{k+1}} G^{\overline{Z}_{-j}}(Z_j,\phi) \ s.t.\ Z_j/\mu_1 + \phi \leq \omega_1.$$

it follows from a standard revealed preference argument that

$$\widetilde{G}^{\overline{Z}_{-j}}(C(Z_j^{\overline{Z}_{-j}*}(q_j^Z;\widetilde{E}(p_x^1,p_h),1)),\phi^{\overline{Z}_{-j}*}(q_j^Z,\widetilde{E}(p_x^1,p_h),1))$$

$$\geq \widetilde{G}^{\overline{Z}_{-j}}(Z_j^{\overline{Z}_{-j}*}(q_j^Z;\widetilde{E}(p_x^2,p_h),1))/\mu_1 + (1-t_w^2)]\omega_1.$$

Hence type 1 household prefer staying in 1 than moving to 2. ■

We now turn to the question of the *sufficiency* of the GSC condition for segregation. As in [**?**] or [2], doing this analysis requires some knowledge of the households preferences defined in the space of all parameters that affect its choice of place of residence, preferences that are described, as mentioned earlier, by the conditional indirect utility function defined in (3). Under homothetic separability, we know from (6) that we can write this conditional indirect utility function $V^{\mathbf{Z}}$ as:

$$V^{\mathbf{Z}}(1,q_h,\omega_i(1-t_\omega)) = G(\mathbf{Z},\frac{\omega_i(1-t_\omega)}{\widehat{E}(q_h)})$$

where $\widehat{E}(q_h) = \overline{E}(1,q_h)$. Suppose now that we focus on some public good j and that we fix the quantities $\overline{\mathbf{Z}}^{-j} \in \mathbb{R}_+^{k-1}$ of the other $k-1$ public goods. We can then represent a typical indifference curve of a household of wealth ω_i in the space $[0,1]\times\mathbb{R}_+$ of all combinations of wealth tax rate and public good j by means of the implicit function $z^I:[0,1]\to\mathbb{R}$ defined by

$$G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}(z^I(t,\omega_i),\frac{\omega_i(1-t)}{\widehat{E}(q_h)}) \equiv a \tag{31}$$

for some a. Since $G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}$ is a twice differentiable concave and strictly increasing function of its two arguments, it is clear that the implicit function z^I is well-defined and twice differentiable. Differentiating (31) with respect to t yields:

$$\partial z^I(\overline{t}, \overline{\omega}_i)/\partial t \equiv \frac{\omega_i}{\widehat{E}(q_h)\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}} \tag{32}$$

If we now differentiate (32) with respect to ω_i, we obtain:

$$\frac{\partial^2 z^I(\overline{t}, \overline{\omega}_i)}{\partial t \partial \omega_i} \equiv \frac{\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{(1-\overline{t})\omega_i}{\widehat{E}(q_h)[G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})}{\widehat{E}(q_h)[\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}]^2} \tag{33}$$

Notice also that the Marhallian demand for public good j conditional upon the quantities $\overline{\mathbf{Z}}^{-j}$of the other $k-1$ public goods can be defined to be the solution of the following program:

$$\max_{Z_j} G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}(Z_j, \frac{\omega_i - p_Z Z}{\widehat{E}(q_h)}) \tag{34}$$

that is characterized (under our conditions) by the first order condition:

$$\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} \equiv \frac{p_j^Z}{\widehat{E}(q_h)} \tag{35}$$

If we differentiate identity (35) with respect to $\widehat{E}(q_h) = p_X$, we obtain (upon manipulations):

$$\frac{\partial Z^{\overline{\mathbf{Z}}_{-j}*}(p_j^Z; p_X, \omega_i)}{\partial p_X} \equiv \frac{-\frac{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{\widehat{E}(q_h)}[\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{\omega_i - p_Z Z^{\overline{\mathbf{Z}}_{-j}*}(.)}{\widehat{E}(q_h)[G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})]}{G_{Z_jZ_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - \frac{2p_j^Z}{\widehat{E}(q_h)} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} + \frac{(p_j^Z)^2}{[\widehat{E}(q_h)]^2} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} \tag{36}$$

If the generalized GSC condition holds, the sign of the left hand side of identity (36) is the same for all values of $(p_j^Z; p_X, \omega_i)$ and all quantities $\overline{\mathbf{Z}}^{-j}$ of the other public goods. As the denominator of the right hand side of (36) is negative thanks to the second order condition of program (34), the sign of the right hand sign is completely determined by the sign of $\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{\omega_i - p_Z Z^{\overline{\mathbf{Z}}_{-j}*}(.)}{\widehat{E}(q_h)[G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})]$ which must be constant under the GSC condition. Because the sign of $\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{(1-\bar{t})\omega_i}{\widehat{E}(q_h)[G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})$ is also what determines the change in the slope of indifference curves as given by (32) brought about by a change in the household wealth, we therefore have that the slope of this implicit function evaluated at any given (t, z) is monotonic with respect to ω_i. This monotonicity property, which implies that any two indifference curves belonging to households with different wealth can cross at most once, will play a key role in the proof of proposition 2 below. It is illustrated in figure 2 below.

itbpFUX306.375pt204.25pt0ptFigure 2Plot;linecolor "black";linestyle 1;pointstyle "point";linethickness 3;lineAttributes "Solid";var1range "0,1";var2range "0,5";num-x-gridlines 24;num-y-gridlines 24;curveColor "[flat::RGB:0000000000]";curveStyle "Line";rangeset"XY";function 61z=exp(−1+2t)1;*linecolor"blue";linestyle1;pointstyle"point";line*
x−gridlines24;num−y−gridlines24;curveColor"[flat :: RGB : 0x000000ff]";curveStyle"Line";r
x−gridlines24;num−y−gridlines24;curveColor"[flat :: RGB : 0x00008000]";curveStyle"Line";pl
properties"XPR";

Using this important single-crossing property, we now establish that, if we restrict attention to economies in $\mathbb{D}^A$ in which households have additively separable preferences, or if we assume that there is only one local public good, then the GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure.

.

Proposition 2 *If households preferences satisfy the generalized GSC condition, then any stable jurisdiction structure associated to an economy in $\mathbb{D}^A$, or to an economy in $\mathbb{D}$ if $k = 1$, is wealth-segregated.*

Proof. Consider first any economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ in $\mathbb{D}^A$ and a jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l\in\mathbb{L}})$ for this economy and denote by q^l the after-tax dwelling price in jurisdiction l defined by $q^l = p^l(1 + t_h^l)$. Proceed by contraposition and assume that the jurisdiction structure is not wealth-segregated. This means that there are households a, b and c in $[0, 1]$ endowed with private wealth $\omega_a < \omega_b < \omega_c$, and 2 jurisdictions l and l', with $a, c \in j^l$ and $b \in j^{l'}$. Either this jurisdiction structure is not stable, and the proof is over, or it is stable. If it is stable than one must have (exploiting the additive separability of the preferences):

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_a) \geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1 - t_\omega^{l'})\omega_a) \quad (37)$$
$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_b) \leq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1 - t_\omega^{l'})\omega_b) \quad (38)$$
$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_c) \geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1 - t_\omega^{l'})\omega_c) \quad (39)$$

where for some continuous and increasing function $\Gamma : \mathbb{R}_+ \to \mathbb{R}$ with at least one inequality being strict (to avoid universal indifference). Suppose that

$q^l \geq q^{l'}$ (the proof being symmetric if $q^l < q^{l'}$. Since both the functions Γ and $\widehat{E}$ are continuous and increasing, we know from the intermediate value theorem that there exists some $\bar{t}_\omega \in [0;1]$ such that $\Gamma(\widehat{E}(q^l)(1-t^l_\omega)) = \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega))$ It follows that:

$$\begin{aligned} g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_a) &= g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_a) && (40) \\ g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_b) &= g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_b) && (41) \\ g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_c) &= g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_c) && (42) \end{aligned}$$

Using the regularity condition on preferences, let $\bar{Z}_j$ be the amount of some public good j such that $g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) = g(\mathbf{Z}^l)$. Hence one has:

$$\begin{aligned} g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_a) &= g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_a) \\ g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_b) &= g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_b) \\ g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_c) &= g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_c) \end{aligned}$$

and, as a result, inequalities (37)-(39) write:

$$\begin{aligned} g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_a) &\geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1-t^{l'}_\omega)\omega_a) && (43) \\ g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_b) &\leq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1-t^{l'}_\omega)\omega_b) && (44) \\ g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_c) &\geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1-t^{l'}_\omega)\omega_c) && (45) \end{aligned}$$

which violates the single-crossing implication of the generalized GCS condition in the plane of all housing tax rates and quantity of public good j. We leave to the reader the task of verifying that the same argument can be established under separability only if there is only one public good. ■

Additive separability (along with regularity) plays a key role in the proof if the number of public good is larger than one. We further emphasize this by providing an example of an economy in $\mathbb{D}$ (but not in $\mathbb{D}^A$) where a stable jurisdiction structure can be non-segregated even when the GSC condition holds. Hence, the GSC condition is not sufficient for segregation if there are several public goods and if preferences are homothetically separable - but not additively so:

Example 1 *Consider and economy $(\omega, U, \mathbb{L}, \{L^l\}_{l\in\mathbb{L}})$ in $\mathbb{D}$ where the households preferences are represented by the utility function:*

$$U(Z_1, Z_2, x, h) = Z_1 + Z_2 \ln(1 + 2(xh)^{\frac{1}{2}})$$

Such an utility function is continuous, increasing and strictly-quasi concave with respect to all its arguments. Furthermore, the function is homothetically separable - but not additively separable - between the 2 public goods on one hand and the two private goods on the other hand. Consequently,using the *two-step budgeting procedure, maximizing the utility function subject to the budget constraint is equivalent to solving the program:*

$$\max_{(Z_1;Z_2;\phi)\in\mathbb{R}^3_+} Z_1 + Z_2 \ln(1+\phi) \ s.\ t.\ p_1^Z Z_1 + p_2^Z Z_2 + p_\phi \phi \leq R \tag{46}$$

where $p_\phi = \overline{E}(p_x, p_h)$. *For any amount* $\bar{Z}_2$ *of public good* 2, *the marshallian demand for* Z_1 *is given by:*

$$\begin{aligned} Z_1^{MC}(p_1^Z, p_\phi, R) &= \frac{R+p_\phi}{p_1^Z} - \bar{Z}_2 \ if \ \frac{R+p_\phi}{p_1^Z} > \bar{Z}_2 \\ Z_1^{MC}(p_1^Z, p_\phi, R) &= 0 \ otherwise \end{aligned}$$

which is always (weakly) increasing with respect to p_ϕ. *Even though it is difficult to provide an explicit definition of the marshallian demand for public good 2 (equal to the conditional demand for that public good thanks to additive separability between the two public goods), we can prove that it is always decreasing with respect to* p_ϕ. *Indeed, the Marshallian demand for public good 2 conditional upon public good 1 is the solution of the following program*

$$\max_{Z^2} Z_2 \ln\left(\frac{p_\Phi + R - p_2^Z Z_2}{p_\Phi}\right) \tag{47}$$

and is characterized therefore by the 1st order condition:

$$\ln\left(\frac{p_\Phi + R - p_2^Z Z_2^M(Z_2;.)}{p_\Phi}\right) - \frac{p_2^Z Z_2^M(Z_2;.)}{p_\Phi + R - p_2^Z Z_2^M(Z_2;.)} \equiv 0$$

Differentiating this identity with respect to p_Φ *and rearranging yields:*

$$\frac{\partial Z_2^M(Z_2;.)}{\partial p_\phi} = \frac{\frac{p_\Phi}{p_\Phi+R-p_2^Z Z_2^M(Z_2;.)}\left[\frac{R-p_2^Z Z_2^M(Z_2;.)}{p_\Phi^2}\right] + \frac{p_2^Z Z_2^M(Z_2;.)}{(p_\Phi+R-p_2^Z Z_2^M(Z_2;.))^2}]}{\frac{-p_2^Z}{p_\Phi+R-p_2^Z Z_2^M(Z_2;.)}\left[2 + \frac{1}{(p_\Phi+R-p_2^Z Z_2^M(Z_2;.)}\right]} \leq 0 \tag{48}$$

Hence, the Marshallian demand for public good 2 conditional on public good 1 is decreasing with respect to the price of the private good so that the generalized GSC condition holds Let us construct a stable but yet non-segregated jurisdiction. For this sake, consider a jurisdiction structure with jurisdictions j^1 *and* j^2 *where* $Z_1^1 = 1/100$, $Z_2^1 = 2$, $p^1 = 1$, $t_h^1 = 0$ *and* $t_\omega^1 = 7/10$, *and* j^2, *by* $Z_1^2 = 0$, $Z_2^2 = 1$, $p^2 = 1$, $t_h^2 = 0$, *and* $t_\omega^2 = 1/1000$, *and 3 types of households* a, b, c *with* $\omega_a = 0,1$, $\omega_b = 1$ *and* $\omega_c = 10$. *Assume also that* $C_1^1 = C_2^1 = C_1^2 = C_2^2 = C$ *with* $C(x) = x$. *Households of types* a *and* c *will prefer to live in jurisdiction* 1 *while households of type* b *will be better-off in jurisdiction* 2. *Indeed, their utility would be (approximately):*
0.011 in j^1 *and 0.001 in* j^2 *for households of type* a,
0,535 in j^1 *and 0.693 in* j^2 *for households of type* b,

2,783 inj^1 *and 2,397 in* j^2 *for households of type c.*
There is obviously no difficulty in finding land endowments and mass μ_a*,* μ_b *and* μ_c *of these households such that* $7\mu_c = 1/100 + 2$ *and* $\frac{\mu_b}{1000} = 1$ *and such that that the demand of land by each of these households equals the available amount of land at these land prices.*

3 Conclusion

The main lesson of this paper holds in one sentence. If a continuum of households with differing wealth but with the same regular and homothetically additively separable preferences for local public goods, private spending and housing are free to choose their favorite combination of dwelling tax rates, wealth tax rates and local public good provision, then any stable jurisdiction structure that results from such a free choice will involve perfect wealth stratification of those households if and only if the households preferences satisfies a generalization of the GSC condition of [**?**]. Yet, as illustrated by the example, the generalization of the GSC condition is not sufficient to ensure the segregation of any stable jurisdiction structure if there is more than one public good if preferences are homothetically separable, but not additively so. While we believe this main lesson to be of some interest, it is clear that more work needs to be done to understand the extent to which this GSC condition is necessary or sufficient for segregation in the case where households preferences are not homothetically separable. It would also be important to test whether the GSC condition is actually verified by households who populate the jurisdictions of the real world. The fact that we have provided such condition in a model with competitive land market opens up the way for empirical testing using the housing market. We plan to provide these empirical tests in our future work.

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