

# The effect of a local allowance on the endogenous formation of jurisdictions

Remy Oddou\*

August 2, 2017

## Abstract

This paper analyses the effect of an allowance, which amount depends on the jurisdiction, on the segregative properties of endogenous formation of jurisdictions. Households choosing to live at the same place form a jurisdiction which aim is to produce a local public good and to implement a redistribution policy, by granting every household an allowance which amount is determined by the jurisdiction. In every jurisdiction, the production of the local public services and the allowance are financed through a local tax based on households' wealth. Local wealth tax rates and the level of the allowance are exogenously determined in every jurisdiction. Household are free to leave their jurisdiction for the jurisdiction that would provide them their highest utility. We find that the existence of an allowance mitigates the segregative properties of endogenous jurisdiction formation, as the condition identified by Gravel and Thoron to ensure the segregation of any stable jurisdiction structure remains necessary, but is not sufficient anymore.

*JEL Classification:* C78; D02; H73; R13

*Keywords:* Jurisdictions; Segregation; Allowance

---

\*Economix, University Paris West, 200 av de la Republique, 92001, Nanterre, France, remy.oddou@free.fr

# 1 Introduction

Local jurisdictions have a well-known role : the production of public goods. But, more and more, they choose to have hand in wealth redistribution. Although, in most countries, local taxation is proportional, which prevent from implementing a redistributive taxation policy, jurisdictions have other ways to implement a policy in order to reduce wealth inequality. They can, for instance, provide an allowance to the households.

As highlighted by many economists (see, for instance, [6]), segregation by wealth seems to be expanding phenomenon. Within an urban area, one can observe jurisdictions with mainly poor households, and jurisdictions with very few of them. Tiebout's 1956 article [8] provides a widely known explanation to this phenomenon : in his opinion, households choose their jurisdiction according to a trade-off between local tax rates and amounts of public services provided, which leads every jurisdiction to be homogeneous : households who wants to consume a large amount of public good will live in a jurisdiction where the tax rate and the amount of public good is high, while households who do not particularly enjoy the public good rather stay in a jurisdiction where the tax rate is low. The formation of jurisdictions structure is therefore endogenous, due to the free mobility of households, that can "vote with their feet", that is to say leave their jurisdiction to another one that fits better with their preferences. An important literature dealing with the endogenous jurisdictions formation *à la* Tiebout exists.

Westhoff [9] must be mentioned among the first models based on Tiebout's article. In his model, households can consume a local public good, financed through a local tax on wealth, and only consumed by household living in the jurisdiction, and a composite private good. The main result of his article is that an equilibrium will exist **if** the slopes of individuals' indifference curves in the tax rate-amount of public good space to be ordered by their private wealth. Under this condition, at equilibrium, the jurisdictions structure will be segregated. It is important, though, to notice that this condition is sufficient, but not necessary.

Gravel and Thoron [5] identified a necessary and sufficient condition to have every stable jurisdiction structure segregated, within Westhoff's meaning. This condition, known as the Gross Substitutability/Complementarity (GSC) condition, requires that the public good must be, for all level of prices and wealth, either always a complement or always a substitute to the private good. In their model, this condition is equivalent to have the favorite tax rate (i.e. the tax rate that maximizes the utility of a certain households, in a certain jurisdiction) being a monotonous function of the private wealth, for any level of prices and wealth.

Biswas, Gravel and Oddou [1] integrated a welfarist central government that maximize a generalized utilitarian social welfare function by implementing an equalization payment policy. Equalization payment can be either vertical (the government taxes households and redistributes the revenues to jurisdictions), horizontal (the government redistributes local tax revenues between the jurisdictions), or mixed. They found that the GSC condition remains necessary and sufficient. However, their model only allows for redistribution among jurisdictions, not among households.

Oddou [7] examined the robustness of this condition when the public good may suffer from congestion, and households may benefit from other jurisdictions' local

public services. In such a framework, the GSC condition is affected neither by the existence of spillovers across jurisdictions, nor the congestion (at least if it is not too strong). Those articles prove the robustness of the GSC condition to several generalizations of Gravel and Thoron's model.

However, as proven by Gravel and Oddou ([4]), two generalizations of the basic model mitigate the segregative properties: the existence of different kinds of public goods within each jurisdiction, and the presence of a housing market (if the preferences over the private good and the housing are not homothetic). These two elements make the GSC condition not sufficient to ensure segregation.

In this article, we start from Gravel and Thoron model, and we assume that jurisdictions can give their households an allowance. The amount of the allowance is the same for every household, and is exogenously fixed by the jurisdiction, so is the tax rate and the amount of public good. One can suppose that it is determined by a vote, or according to a welfare function. Jurisdictions' budget must be balanced, so the production cost of the public good added to the allowance must be equal to the tax revenue.

This framework can be seen as a taxation that is progressive instead of proportional, because the share of their gross wealth that is taxed is increasing with respect to the wealth. In many countries, though, local taxes are more or less proportional to the revenue, because local taxes are based on the housing value (and a fixed rate), and the share of their income that households devote to housing does not depend on the income (see [2]). However, some countries have implemented or are implementing a progressive local taxation.

Households are assumed to be freely mobile, so, once all jurisdictions have determined their tax rate, the amounts of public goods and allowance, households can leave their jurisdiction for another one that would increase their utility. Equilibrium is reached when no household has an incentive to leave unilaterally its jurisdiction.

We prove that, in the presence of jurisdictional allowance, the monotonicity of the favorite tax rate function remains necessary to have every stable jurisdiction structure segregated, but the GSC is no longer sufficient to ensure wealth-stratification. Furthermore, the GSC condition is not equivalent to the monotonicity of the favorite tax rate function.

If the public good is a gross complement of the private good, then, at equilibrium, the jurisdiction structure may look like the situation defined by Epple and Romano [3] as "the ends against the middles": on one hand, poor and rich households rather live in jurisdictions with a high tax rate, the first because they can benefit from a high allowance, the second because they can enjoy a high amount of public good, that is a complement to the private good. The intermediate households, on the other hand, prefer a jurisdiction with a low tax rate.

This paper aims at examining the segregative properties of the endogenous jurisdiction structure formation in such a framework. The article is organized as follows. The next section introduces the formal model. Section 3 provides an example of how congestion and spillovers can modify a jurisdiction structure. Section 4 states and proves the results. Finally, section 5 concludes.

## 2 The formal model

We consider a model with a continuum of households on the interval  $I$  with Lebesgue measure  $\lambda$ , where, for any subset  $I_0 \subset I$ , the mass of household in  $I_0$  is given by  $\lambda(I_0)$ . Households' wealth distribution is given by a Lebesgue measurable function  $\omega : I \rightarrow \mathbb{R}_+^*$  - household  $i$  is endowed with a wealth  $\omega_i \in \mathbb{R}_+$  - with  $\omega$  being an increasing and bounded from above function.

All households share identical preferences, represented by a twice differentiable, increasing and concave utility function

$$U : \begin{array}{l} \mathbb{R}_{++}^2 \longrightarrow \mathbb{R}_+ \\ (Z, x) \longmapsto U(Z, x) \end{array}$$

where

1.  $Z$  is the available amount of public good,
2.  $x$  is the amount of a composite private good.

We denote  $Z^M(p_Z, p_x, R)$  and  $x^M(p_Z, p_x, R)$  the Marshallian demands for the public good and the private good (respectively), when the public good price is  $p_Z$ , the private good price,  $p_x$ , and the revenue,  $R$ . We also define  $MRS(Z, x)$  as the Marginal Rate of Substitution of the public good to the private good.

We assume that both the public good and the private good are normal goods, which means that their Marshallian demands are increasing with respect to the revenue.

$\mathbb{U}$  represents the set of all functions satisfying the properties defined above.

We can now define the Marginal Rate of Substitution (MRS). as the ratio of the derivative of the utility function with respect to the public good over the derivative of the utility function with respect to the private good.

**Definition 1.** *The Marginal Rate of Substitution is the ratio of the derivative of the utility function with respect to the public good over the derivative of the utility function with respect to the private good:  $MRS(Z, x) = \frac{\frac{\partial U(Z, x)}{\partial Z}}{\frac{\partial U(Z, x)}{\partial x}}$*

Households choose their place of residence among the finite set of locations, represented by  $\mathbb{L} \subset \mathbb{N}$ . The possibility for some locations to be empty is allowed. Households living at the same location form a jurisdiction. We denote  $J \subseteq \mathbb{L}$  the set of jurisdictions, and  $I_j \subseteq I$  as the subse of households that lives in jurisdiction  $j$ . As households must live in one, and only one, jurisdiction, one has  $\bigcup_{j \in J} I_j = I$  and,

$$\forall (j, j') \in J^2, I_j \cap I_{j'} = \emptyset.$$

Jurisdictions have two purposes : producing a public good, that will be consumed only by the households that composed it, and distributing an allowance to households. To finance those two functions, the jurisdictions raise a tax proportional to households' wealth. The tax rate in jurisdiction  $j$  is denoted  $t_j$ , the amount of allowance

provided by jurisdiction  $j$  is denoted  $G_j$ .

The amount of local public good produced by jurisdiction  $j$  is given by

$$Z_j = t_j \varpi_j - \mu_j G_j$$

with :

- $\mu_j = \lambda(I_j)$  being the mass of households in  $j$ ,
- $\varpi_j = \int_{i \in I_j} \omega_i$  being the aggregated wealth in  $j$ .

The amount of the composite private good that household  $i$  living in jurisdiction  $j$  is given by

$$x_{ij} = (1 - t_j)\omega_i + G_j$$

**Definition 2.** *A economy is composed of 3 elements:*

- A wealth distribution  $\omega$
- Preferences represented by the utility function  $U \in \mathbb{U}$
- A set of location  $\mathbb{L} \in \mathbb{N}$

We denote  $\Delta$  as the set of all conceivable economies. We must now define the notion of jurisdiction structure.

**Definition 3.** *A jurisdiction structure is a vector:*

$$\Omega = (J, \{I_j\}_{j \in J}; (\{t_j\}_{j \in J}); (\{G_j\}_{j \in J}))$$

In words, a jurisdiction structure is characterized by the set of jurisdiction that composes it, the partition of households among the different jurisdictions, and the policy (tax rate and amount for the allowance) implemented by the different jurisdiction.

**Definition 4.** *A jurisdictions structure  $\Omega = (J, (\{I_j\}_{j \in J}); (\{t_j\}_{j \in J}); (\{G_j\}_{j \in J}))$  is **stable** in the economy  $(\omega, U, (a, \alpha), B, \mathbb{L})$  if and only if*

1.  $\forall (j, j') \in J^2, \forall i \in I_j, U(Z_j, (1 - t_j)\omega_i + G_j) \geq U(Z_{j'}, (1 - t_{j'})\omega_i + G_{j'})$ ,
2.  $\forall j \in J, Z_j \leq \frac{t_j \varpi_j - \mu_j G_j}{p_Z}$ .

Literaly, a jurisdictions structure is stable if and only if :

1. No household can increase its utility by modifying its consumption bundle or by leaving its jurisdiction,
2. Every jurisdiction's budget is balanced.

Let us now express formally the definition of the segregation, which is the same definition as in [9].

**Definition 5.** *A jurisdictions structure  $\Omega = (J, (\{I_j\}_{j \in J}); (\{t_j\}_{j \in J}); (\{G_j\}_{j \in J}))$  in the economy  $(\omega, U, \mathbb{L})$  is **segregated** if and only if  $\forall \omega_h, \omega_i, \omega_k \in \mathbb{R}_+$  such that  $\omega_h < \omega_i < \omega_k$ ,  $(h, k) \in I_j$  and  $i \in I_{j'} \Rightarrow Z_j = Z_{j'}, G_j = G_{j'}$  and  $t_j = t_{j'}$*

In words, a jurisdictions structure is wealth-segregated if, except for groups of jurisdictions offering the same available amount of public good, the same amount of allowance and the same tax rate, the poorest household of a jurisdiction with a high per capita wealth is (weakly) richer than the richest household in a jurisdiction with a lower per capita wealth.

In the next section, we examine the robustness of the monotonicity of the favorite tax rate function and the GSC condition to ensure segregation.

### 3 Results

This article proves that the GSC condition is not equivalent anymore to the monotonicity of the favorite tax rate function with respect to the private wealth, and that the monotonicity of the favorite tax rate function remains necessary to have all stable jurisdictions structures segregated, but is not sufficient anymore, nor is the GSC condition. Let us define the GSC condition first.

**Definition 6.** *If the GCS condition holds, then, one has either  $\frac{\partial Z^M(p_Z, p_x, R)}{\partial p_x} \leq 0 \forall (p_Z, p_x, R)$  (if  $Z$  is a gross complement to  $x$ ) or  $\frac{\partial Z^M(p_Z, p_x, R)}{\partial p_x} \geq 0 \forall (p_Z, p_x, R)$  (if  $Z$  is a gross substitute to  $x$ )*

We now define the favorite tax rate function.

**Definition 7.**  $\forall (\varpi, \mu, G, \omega_i) \in \mathbb{R}_+^4$ , we define

$$t^* : \begin{cases} \mathbb{R}_+^4 & \longrightarrow [0; 1] \\ (\varpi, \mu, G, \omega_i) & \longmapsto t^*(\varpi, \mu, G, \omega_i) = \underset{t \in [0; 1]}{\operatorname{argmax}} U(t\varpi - \mu G), (1-t)\omega_i + G \end{cases}$$

as the favorite tax rate function, eg the tax rate that maximizes the utility of a household endowed with private wealth  $\omega_i$ , in a jurisdiction with an aggregate wealth  $\varpi$ , a mass of population equal to  $\mu$ , that grants an amount  $G$  of allowance.

According to the assumptions made on the utility function, this "favorite tax rate function" always exists. For  $G = 0$  (as in Gravel and Thoron's article), the monotonicity of the favorite tax rate function is equivalent to the GSC condition, but it is not the case otherwise. However, the next lemma will define the relation that exists between the favorite tax rate function and the Marshallian demand for the public good.

Some conditions have to be respected for  $t^*$  to exist. For instance, clearly, the function will not exist if  $\varpi < \mu G$ , because even if all the wealth was taxed, it would not be sufficient to finance the allowance.

Let us define the following function, representing the utility with respect to the tax rate:

$$\Phi : \begin{cases} [0; 1] \times \mathbb{R}_+^4 & \longrightarrow \mathbb{R}_+ \\ (t; \varpi, \mu, G, \omega_i) & \longmapsto \Phi(t; \varpi, \mu, G, \omega_i) = U(t\varpi - \mu G), (1-t)\omega_i + G \end{cases}$$

As the utility function is assumed to be concave, the function  $\Phi(t; \varpi, \mu, G, \omega_i)$  is single-peaked with respect to  $t$ , i.e.  $\forall \bar{t} < t^*(\varpi, \mu, G, \omega_i)$  (resp.  $\bar{t}$ ),  $\forall t \in ]\bar{t}; t^*(\varpi, \mu, G, \omega_i)[$

(resp.  $\forall t \in ]t^*(\varpi, \mu, G, \omega_i); \bar{t}[$ ,  $\Phi(\bar{t}; \varpi, \mu, G, \omega_i) < \Phi(t; \varpi, \mu, G, \omega_i) < \Phi(t^*(\varpi, \mu, G, \omega_i); \varpi, \mu, G, \omega_i)$ ).

The next lemma will define the relation between the favorite tax rate function and the Marshallian demand for the public good.

**Lemma 1.** *For any preferences belonging to  $\mathbb{U}$ , one has,  $\forall (\omega_i, \varpi, \mu, G) \in \mathbb{R}_+^4$ ,*

$$t^*(\varpi, \mu, G, \omega_i) \equiv \frac{Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{\mu}{\varpi})) + \mu G}{\varpi} \quad (1)$$

*Proof.* At the optimum, the MRS is equal to the price ratio. Hence, one has:

$$MRS = \frac{p_Z}{p_x} \quad (2)$$

The first order condition (FOC) implies that:

$$\frac{U_Z(t^*\varpi - \mu G; (1 - t^*)\omega_i + G)}{U_x(t^*\varpi - \mu G; (1 - t^*)\omega_i + G)} = \frac{\omega_i}{\varpi} \quad (3)$$

Consequently, combining 2 and 3, we know that:

$$t^*(\varpi, \mu, G, \omega_i)\varpi - \mu G = Z^M(p_Z, p_x, R)(1 - t^*(\varpi, \mu, G, \omega_i)\omega_i + G) = x^M(p_Z, p_x, R) \quad (4)$$

when  $p_Z = \frac{1}{\varpi}$ ,  $p_x = \frac{1}{\omega_i}$  and  $R = 1 + G(\frac{1}{\omega_i} + \frac{\mu}{\varpi})$ . which leads to the result.  $\square$

Thanks to this lemma, we can observe that, contrary to Gravel and Thoron's article, the GSC condition will not be equivalent to the monotonicity of the favorite tax rate function with respect to the private wealth<sup>1</sup>.

However, the previous lemma allows us to identifies the following implication.

**Lemma 2.** *If the public good is a gross substitute of the private good, then the favorite tax rate function is decreasing with respect to the private wealth.*

*Proof.* Deriving 1 with respect to  $\omega - i$ , one has:

$$\frac{\partial t^*(\varpi, \mu, G, \omega_i)}{\partial \omega_i} = \frac{-1}{p_x^2 \varpi} \left[ \frac{\partial Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{\mu}{\varpi}))}{\partial p_x} + G \frac{\partial Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{\mu}{\varpi}))}{\partial R} \right] \quad (5)$$

As  $\frac{\partial Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{\mu}{\varpi}))}{\partial p_x} > 0$  if the public good is a gross substitute to the private good, and  $\frac{\partial Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{\mu}{\varpi}))}{\partial R} > 0$  because the public good is normal, then the favorite tax rate function will be decreasing **if** the public good is a gross substitute.  $\square$

We can now formally state the main result of the article.

**Theorem 1.** *For all economies belonging to  $\Delta$ , the monotonicity of the favorite tax rate function with respect to the private wealth is necessary but not sufficient to have any stable jurisdiction structure segregated.*

<sup>1</sup>If  $G = 0$ , as in Gravel and Thoron's article, the GSC condition is equivalent to the monotonicity of the favorite tax rate function with respect to the private wealth, but this results is not valid anymore if  $G > 0$ .

Let us start by the necessity part of the theorem.

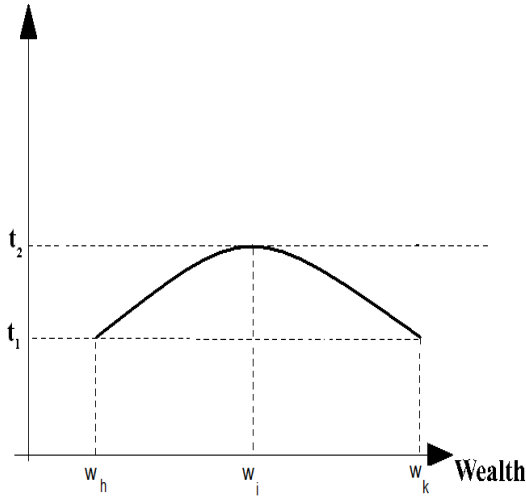
**Proposition 1.** *For all economies belonging to  $\Delta$ , the monotonicity of the favorite tax rate function with respect to the private wealth is necessary to ensure the segregation of any stable jurisdiction structure.*

*Proof.* To prove this proposition, we show that any violation of the condition allows to construct a stable and yet non segregated jurisdiction structure.

Let us suppose that the monotonicity of the favorite tax rate function with respect to the private wealth condition is violated, for some a non-degenerated interval of private wealth  $[\omega_1; \omega_3]$ , for some aggregate wealth  $\bar{\omega}$ , for some amount of allowance  $\bar{G}$  and some mass of households  $\bar{\mu}$ . Let suppose, with no loss of generality, that the favorite tax rate function is increasing on the interval  $[\omega_1; \omega_b]$  and decreasing on the interval  $[\omega_b; \omega_3]$ ,  $\omega_b \in ]\omega_1; \omega_3[$ .

Consequently, we know that there exist  $\omega_a < \omega_b$  and  $\omega_c > \omega_b$  such that  $t^*(\bar{\omega}, \bar{\mu}, \bar{G}, \omega_a) = t^*(\bar{\omega}, \bar{\mu}, \bar{G}, \omega_c) = t_1 < t_2 = t^*(\bar{\omega}, \bar{\mu}, \bar{G}, \omega_b)$ .

### Favorite tax rate



Let us denote  $Z_{j=\{1,2\}} = t_j \bar{\omega} - \bar{\mu} \bar{G}$  and  $x_{j=\{1,2\}}^{i=\{a,b,c\}} = (1 - t_j) \omega_i + \bar{G}$ . According to the definition of the favorite tax rate function, one has:

- $U(Z_1, x_1^1) > U(Z_2, x_2^1)$
- $U(Z_1, x_1^2) < U(Z_2, x_2^2)$
- $U(Z_1, x_1^3) > U(Z_2, x_2^3)$

Let consider 3 types of households,  $p, m$  and  $r^2$ , such that households of type  $p$  have no wealth, households of type  $m$  are endowed with a wealth equal to  $\omega_c$ , and households of type  $r$ , to  $\frac{\omega_c(\omega_c - \omega_a)}{\omega_b - \omega_a}$ . Clearly, one has  $\omega_p < \omega_m < \omega_r$ .

---

<sup>2</sup> $p$  is for poor,  $m$  for middle and  $r$  for rich.



Let us place a mass  $\mu_p = \epsilon$  of households  $p$  and a mass  $\mu_r = \frac{(Z_1 + \epsilon x_1^1)(\omega_b - \omega_a)}{\omega_c(\omega_c - \omega_a - (1-t_1)(\omega_b - \omega_a))}$  of households  $r$  in a jurisdiction  $\alpha$  with  $t_\alpha = 1 - \frac{(1-t_1)(\omega_b - \omega_a)}{\omega_c}$ ,  $G_\alpha = x_1^1$ , and, thus,  $\varpi_\alpha = \frac{(\omega_c - \omega_a)(Z_1 + \epsilon x_1^1)(\omega_b - \omega_a)}{(\omega_b - \omega_a)(\omega_c - \omega_a - (1-t_1)(\omega_b - \omega_a))}$ . Such a jurisdiction would produce an amount of public good equal to  $Z_1$ , and would respectively provide to households  $a, b$  and  $c$  an amount of private good equal to (respectively)  $x_1^1, x_1^2$  and  $x_1^3$ .

Let us now place a mass  $\mu_b = \frac{Z_2}{\omega_c - (1-t_2)\omega_b}$  of households  $b$  in a jurisdiction  $\beta$  with  $t_\beta = 1 - \frac{(1-t_2)(\omega_i - \omega_h)}{\omega_k}$ ,  $G_\beta = x_2^1$ , and, therefore,  $\varpi_\beta = \frac{\omega_k Z_2}{\omega_c - (1-t_2)\omega_b}$ . Such a jurisdiction would produce an amount of public good equal to  $Z_2$ , and would respectively provide to households  $a, b$  and  $c$  an amount of private good equal to (respectively)  $x_2^1, x_2^2$  and  $x_2^3$ .

This jurisdiction structure would therefore be stable and non segregated, which proves the proposition.  $\square$

**Proposition 2.** *For all economies belonging to  $\Delta$ , the GSC is not sufficient to ensure the segregation of any stable jurisdiction structure.*

*Proof.* To prove this proposition, we pick two utility functions, the public good being a gross complement to the private good with the first one, and a gross substitute for the second one, and, for each preferences, we construct a stable and yet non-segregated jurisdiction structure.

Let us consider the following utility function :  $U(Z, x) = \ln(Z) - \frac{1}{x}$

Such an utility function is continuous, twice differentiable, strictly increasing and concave with respect to every argument. The public good is a gross complement to the private good, as the Marshallian demand for the public good is given by:

$$Z^M(p_Z, p_x, R) = \frac{2R + p_x - \sqrt{p_x^2 + 4Rp_x}}{2p_Z}$$

which is decreasing with respect to  $p_x$ . Consider an economy with two jurisdictions  $j_1$  and  $j_2$  and three types of households  $a, b, c$  with private wealth  $\omega_a = 1$ ,  $\omega_b = 2$  and  $\omega_c = 20$ , with  $n_a = 62$ ,  $n_b = 34$  and  $n_c = 1$ .

A stable jurisdiction structure would be households of type  $a$  and  $c$  living in one jurisdiction, denoted  $j_1$ , with  $G_1 = 0.4$  and  $t_1 = 0.78$ , hence one has  $Z_1 = 38.76$ , and households of type  $b$ , living in a second jurisdiction  $j_2$ , with  $G_2 = 0$  and  $t_2 = 0.5$ , hence one has  $Z_2 = 34$ .

Consequently, one has the following utility level (rounded to two digits after the decimal point):

	$j_1$	$j_2$
a	2.04	1.53
b	2.47	2.53
c	3.45	3.43

Hence, the above jurisdiction structure is stable and yet, non-segregated.

Let us now consider the same economy, but with an utility function for which the public good is a gross substitute of the private good, such as:  $U(Z, x) = \ln(Z) + \sqrt{x}$

This function is also continuous, twice differentiable, strictly increasing and concave with respect to every argument. The Marshallian demand for the public good is given by :

$$Z^M(p_Z, p_x, R) = \frac{2(\sqrt{p_x^2 + Rp_x} - p_x)}{p_Z}$$

which is increasing with respect to  $p_x$

We can construct a stable jurisdiction structure by placing households of type  $a$  and  $c$  in jurisdiction  $j_1$ , with  $G_1 = 0.1$  and  $t_1 = 0.48$ , hence one has  $Z_1 = 33.06$ , and households of type  $b$ , living in a second jurisdiction  $j_2$ , with  $G_2 = 0$  and  $t_2 = 0.5$ , hence one has  $Z_2 = 34$ .

One will, therefore, have the following utility level (rounded to two digits after the decimal point):

	$j_1$	$j_2$
a	4.29	4.23
b	4.57	4.53
c	6.74	6.69

Hence, the above jurisdiction structure is also stable and yet, non-segregated, which proves the proposition. □

Using this proposition, we can state that the monotonicity of the favorite tax rate function with respect to the private wealth is not sufficient neither, because, thanks to lemma 2, we know that the favorite tax rate function will be increasing with respect to the private wealth **if** the public good is a gross substitute to the private good. Consequently, if the gross substitutability of the public good is not sufficient, then the monotonicity of the wealth is not sufficient neither.

## 4 Conclusion

The conclusion of this paper is that the presence of an allowance mitigates the segregative properties of endogenous jurisdiction formation, as the GSC condition, that was proven to be robust to several generalization of the model, is not sufficient anymore to ensure the segregation of any stable jurisdiction structure, neither is the monotonicity of the favorite tax rate function with respect to the private wealth.

Also, either proving that the condition is still valid with strong congestion effects, or on the contrary creating a counter-example of an economy violating the condition and such that any stable jurisdictions structure is segregated would be an interesting challenge.

## References

- [1] R. Biswas, N. Gravel, and R. Oddou. The segregative properties of endogenous jurisdiction formation with a welfarist central government. *Social choice and Welfare*, 2012. available online.

- [2] M. A. Davis and F. Ortalo-Magné. Households expenditures, wages, rents. *Review of Economics Dynamics*, 2010. in press.
- [3] D. Epple and R. Romano. Ends against the middle: Determining public service provision when there are private alternatives. *Journal of Public Economics*, 62, 1996.
- [4] N. Gravel and R. Oddou. The segregative properties of endogenous jurisdiction formation with a land market. *Journal of Public Economics*, 117:15–27, 2014.
- [5] N. Gravel and S. Thoron. Does endogeneous formation of jurisdictions lead to wealth stratification ? *Journal of Economic Theory*, 132:569–583, 2007.
- [6] J. Madden. *Problem of the century: racial stratification in the United States*. E. Anderson and D. Massey, New York, NY, 2004.
- [7] Oddou R. The effect of congestion and spillovers on the segregative properties of endogenous jurisdiction structure formation. *Journal of Public Economic Theory*, 2016.
- [8] C. M. Tiebout. A pure theory of local expenditures. *Journal of Political Economy*, 64:416–424, 1956.
- [9] F. Westhoff. Existence of equilibria in economies with a local public good. *Journal of Economic Theory*, 14:84–102, 1977.