Endogenous jurisdictions formation and its segregative properties: a survey of the literature

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Abstract

This paper provides a survey of the literature on human regroupings based on voluntary participation, with a focus on the theoretical and empirical articles dealing with the endogenous processes of jurisdictions formation and its segregative properties. We start from Tiebout's intuitions: households reveal their preferences for the public good by choosing the community according to a trade-off between the tax rate implemented by the different jurisdictions and the amount of public good they provide. Then, the paper analyzes how such intuitions have been modeled in the literature, and reviews the different questions that economists have examined: the existence of an equilibrium, its properties in terms of efficiency and the definition of segregation and its causes.

1 Introduction

According to historians and social scientists, humain regroupings are defined as a "merging of similar individuals who want to live together in order to satisfy some needs such as security and poll their talents so as to make life easier"[1]. The first human regrouping are assumed to have taken place during the Paleolithic.

The first communities were composed of nomadic hunters, that had to move frequently in order to find foods and to survive: in turn, each individual take care of the group while the others rest. Such communities can be considered as Clubs, since the guard is a non-rival and excludable service.

Although some human regroupings could be involuntary (such as esclavagism), or voluntary, but for other motivations than producing agood (for instance, people living close to a river to enjoy the proximity of the water), this paper will

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[1] http://pages.usherbrooke.ca/manuel-histoire/definitions/paleo2.html

exclusively deal with voluntary regrouping which aim is to produce a collective good, that is to say a good that can be equally consumed by anyone that belonged to the community.

As it is well-known in economics, there exist 4 kinds of goods, according to the rivalty and the possibility of exclusion from the consumption of the good:

	Rival	**Non-rival**
Excludable	Private Good	Club good
Non-excludable	Common Ressource	Public good

Gathering is obviously the most efficient way to produce a collective good at a lower cost, since the cost is shared between several individuals. However, the formation of a human group raises several questions: what leads individuals or households to join a group? Will people who choosed to belong to the same group be alike in terms of wealth? in terms of preferences?...

To answer these questions, human beings started to develop societies so as to provides rules to answer those questions. A society, from the Latin word "socius", which means ally, partner, is defined by Joseph Fichter as "a model of organization, institutions and relations between individuals and groups which aim is to concertedly satisfy the collecticvity's needs". Though there exists a wide varieties of societies, this paper will only consider formal local communities having a legal existence: local jurisdictions.

A jurisdiction is a public entity delimited by geographical boundaries, which aim is to provide local public services. Every individual living inside the jurisdiction's boundaries is automatically one of its members. The range of the jurisdictions' competencies is most of the time determined by the central government. For instance, the central government decides whether or not a local jurisdiction is in charge of primary school, and if it transfer this competency to the jurisdiction, then the jurisdiction is free to choose what share of its budget it will devote to primary school. Nowadays, the share of the local public spendings out of the total public spendings can reach 50% in some countries, like the USA. This probably explains why a large body of the literature on local public economics has been developped in this country.

Local public economics is, to some extent, similar to public economics in general, in the sense that it deals with the intervention of a public entity in order to improve an inefficient situation that free market can not handle. Futhermore, most of the time, the voting rule to determine the optimal amount of public good that has to be provided are the same at the national and the local level.

However, local public economics differs from traditional public economics on

two issues:

1. It is easier from a household to move from one jurisdiction to another than leaving its country,

2. The national level can have an impact on local decisions, for instance by implementing a fiscal equalization policy.

The co-existence of different levels of government that produce public goods is called "fiscal federalism". A lot of questions on fiscal federalism have raised: what is the optimal number of levels of political decision-maker? what is their optimal scope? what range of competencies should be decentralized?...

Some of these questions find answers in the literature, in particular in the survey provided by Oates[48]. But his survey focused more on normative questions, while this survey presents the positive results on the endogenous jurisdictions formation and its segregative properties.

The first article dealing with the endogenous formation of jurisdictions was actually a response to Samuelson, provided by Tiebout. In his article [54], Samuelson claimed that voluntary contributions to produce a public good can not be implemented, because households have no incentive to reveal their preferences for the public good, since, by definition, a public good is not excludable (or at a very high cost), hence the benefit a household can enjoy from the public good does not depend on how much it has spend to produce it, so households would have no incentive to reveal their preferences for the public good. Consequently, taxing households on the basis of their preferences, as proposed by Lindhal[45], is not feasible. As a consequence, a public good may not be produced even if the benefits it provided is greater than its cost. Hence, public goods can not be provided through mercantil process based on exlusive ownership, but by an public authority, that finances the production cost with compulsory levies. Consequently, if households' preference can not be observed, the only possibility to have a public good produced is to tax households independently on their preferences over the public good.

As a response, Tiebout [57] asserted that, within any urban area, households can choose their place of residence among several jurisdictions, implementing different tax rate and so offering different quantity of local public good. Contrary to a pure public good, that is non-excludable, a local public good can be consumed only by households living in the jurisdiction that produces it.

In several articles we will explore, local public goods were supposed to be pure local public goods: a local public good does not suffer from congestion and can be consummed only by households living in the jurisdiction that produces it. Consequently, local public goods can be seen as Club goods, as defined by Buchanan[10]: they are non-rival and can be consumed only by members of the

jurisdiction that produces them.

If an empirical article by Heikkila[36] confirmed the assumption that local public goods are club goods, i.e. non-rival and excludable, several articles relaxed it, by considering either that local public good can be partially consumed by households living in other jurisdictions, or that the public good may suffer from congestion effects.

The debate on the congestion indicates that local public goods are not pure club goods, but are in between club goods and private goods. Other economists, such as Nechyba[46] or Bloch and Zenginobuz[9] and [8], as we will see below, consider that local public goods are not totally excludable, which means that households living in a jurisdiction can partially or totally consume other jurisdictions' local public goods, and then are in between club goods and pure public goods.

Except for the case of private government, whose impacts on welfare and on the existence of an equilibrium has been studied by Helsley & Strange[37], a local public good can partially be consumed by households living in other jurisdictions.

In Tiebout's opinion, households, by choosing their place of residence according to a trade-off between the amounts of local public good provided by the jurisdictions and the tax rate they implement, reveal their preferences over the public good, since the choice among the different "tax rate-amount of public good" packages are similar to a choice between several "price paid-amount of good received" combinations.

Tiebout's intuitions must be understood in a context of an urban area with a large number of disctint jurisdictions, implementing different tax rates and offering different amounts of public good. Households are assumed to be able to move from one jurisdiction to another freely, and to have perfect knowledge about the policy implemented by every jurisdiction in the urban area. Under those assumptions, jurisdictions are not formed exogenously, but endogenously by the households that choose their place of residence according to a usual trade-off between a quantity of good and the price to paid to be able to enjoy such quantity. If the number of jurisdiction is large enough, then, for every household, there exists a jurisdiction applying exactly the policy it would pick if it were the dictator.

In his conclusion, Tiebout evoked the homogeneity of the jurisdictions structure: jurisdictions, at equilibrium, will be composed of households having exactly the same preferences over the public good, so the policy will be unanimously determined. Consequently, the endogeneity of the jurisdiction formation seems to have segregative properties, since it leads to assortive matching. However, Tiebout was not very specific about the definition of segregation, since

the main objective of his article was to demonstrate that preferences over the public good can be observed.

Though very influencial, Tiebout's article was not formal at all. It took two decades before a coherent model based on his intuitions could be developped.

Then, a wide literature explores the validity and the consequence of Tiebout's intuitions. First, the preferences over the public good can be observed only at the equilibrium, which raises several question: does an equilibrium always exist? if not, under which conditions is the existence of an equilibrium ensured? Second, the impact of the housing market and of the possible taxation scheme (the public good can be financed through, for instance, a tax on wealth, or through a tax based on the housing value) on the existence of an equilibrium and the segregation has been investigated by many economists. Third, some authors asserts that local public good are not "pure" public good, they may either be partially consumed by households that do not live in the jurisdiction that produce them, or they may suffer from a congestion effect. Finally, a large number of articles investigate on the definition of the segregation, and on the segregative properties of endogenous jurisdictions structure formation.

The next section will present the different attempts at modelling Tiebout's intuitions, using microeconomic theory concepts, and sometimes the cooperative Game Theory tools. The third section will examine the main results on the existence of an equilibrium and its efficiency. The fourth section will focus on the definition of the segregation and its causes. Finally, the fifth section will conclude.

2 Modelling Tiebout's intuitions

2.1 The general approaches

Westhoff [59] was among the first economists to provide a formal model of multijurisdictional economies. The model is simple, there exist 2 goods: a local public good (Z) and a composite private good (x). Every jurisdiction produces a local public good through a tax based on households wealth, that is exogenous. There exist a continuum of households belonging to the interval $[0;1]$, and a set of jurisdictions $J \subset \mathbb{N}, \sharp(J) = M$. A jurisdiction j is composed of a measurable subset $I_j \subset [0;1]$. The number, possibly null, $\int_{I_j} d\lambda$, where λ is the Lebesgue measure, can represent for instance the mass of households that live in jurisdiction j. Every households must choose an unique place of residence, so, $\forall (j,j') \in J, j \neq j', I_j \cap I_{j'} = \emptyset$ and $\bigcup_{j \in J} I_j = [0;1]$. A jurisdiction j is characterized by the tax rate t_j it applies and by the amount of public good it produces (Z_j). Households differ in wealth and in preferences. For simplicity, Westhoff assumes that households are ordered by their wealth: the

households' wealth distribution is modeled as a Lebesgue measurable function $\omega : [0;1] \to \mathbb{R}_+^*$ - household i is endowed with a wealth $\omega_i \in \mathbb{R}_+^*$ - with ω being increasing and bounded from above. Preferences are represented by a function that is continuous, increasing, twice differentiable and quasi-concave with respect to every argument:

$$U_i : \left| \begin{array}{ccc} \mathbb{R}_{++}^2 & \longrightarrow & \mathbb{R}_+ \\ (Z,x) & \longmapsto & U(Z,x) \end{array} \right.$$

Moreover, Westhoff assumed that $\forall i \in [0;1], \forall (Z,Z',x,x') \in \mathbb{R}_{++}^{4*}, U_i(Z,x) > U_i(Z',0)$ and $U_i(Z,x) > U_i(0,x')$. In words, all households would prefer a consumption bundle with strictly positive amounts of the public good and the private good to any bundle with either no public good or no private good.

All the tax revenues are devoted to the production of the public good, that is produced through a linear cost function, so $Z_j = t_j \varpi_j$, where $\varpi_j = \int\limits_{I_j} \omega_i d\lambda$ is the aggregated wealth in jurisdiction j. The budget constraint households must respect is given by $x \leq (1-t)\omega_i$. Since the utility is always increasing with respect to x, and since there is no savings, all the net-of-tax is consumed, so $x = (1-t)\omega_i$. Hence, the utility function can be re-written as follows:

$$U_i(Z,x) = U_i(t\varpi, (1-t)\omega_i)$$

Contrary to Tiebout, Westhoff does not assume that the number of jurisdiction is large enough to have every household living in a jurisdiction that applies exactly the policy that would maximizes its utility. As a consequence, Westhoff integrate a collective decision rule: the majority voting rule.

Using the median voter theorem, this assumption is equivalent to assuming that, in every jurisdiction, the tax rate is such that half of the households living in the jurisdiction would weakly prefer a higher tax rate while half of the households would not be worse-off with a lower tax-rate. Since the utility function is assumed to be concave with respect to each argument, one can prove that preferences are single-peaked over t, so, using Black's median voter theorem[7], we know that there exists $\bar{t}$ such that $\bar{t}$ is preferred by half of the voters to any tax rate lower than $\bar{t}$ and by half of the voters to any tax rate higher than $\bar{t}$. Once each jurisdiction has determined its tax rate, households are free to leave their jurisdiction for the one that would maximizes their utility.

A jurisdiction structure is the specific distribution of households among the different jurisdictions, and a $2M$-vector $(\{t_j, Z_j\}_{j \in J})$ representing the tax rates and the amounts of public goods produced by every jurisdiction. A jurisdiction structure is stable if and only if:

- Every jurisdiction has its budget balanced: $\forall j, Z_j = t_j \varpi_j$,
- In every jurisdiction, the tax rate is the one chosen by the median voter,

- No household has incentive to leave unilaterally its jurisdiction: $\forall j \in J, \forall i \in I_j, U_i(t_j\varpi_j, (1-t_j)\omega_i) \geq U_i(t_k\varpi_k, (1-t_k)\omega_i) \forall k \in J$

This notion of equilibrium is widely used in the literature of local public goods. It is known as the free mobility equillibrium. This notion considers only individual mobility, which is consistent with the assumption of a continuum of households: since every household has a null measure, its decision of leaving unilaterally its jurisdiction could not modify the tax base, nor the voting outcome. While mostly used, this notion is weaker than another definition of stability that consider group deviations (for instance [32] that we will mention later). Once the equilibrium is reached, the jurisdictions structure is stable.

Other economists have provided alternative models for Tiebout intuition. One of them is Ellickson. In his article [21], the set of households is discrete, and there are different private goods and public goods. The main difference is the non-divisibility of the local public goods. Contrary to Westhoff, Ellickson assumed that households could consume either one or zero unit of each public good, and at most one public good.

The fact that households consume a vector of private goods instead of a composite private good is not crucial, since different private goods prices vectors among jurisdictions would have the same effect than different tax rates: the private consumption will vary from one jurisdiction to another.

However, the non-divisibility of the public goods introduces a non-convexity of the consumption set, which, *a priori*, could have caused troubles to prove the existence of an competitive equilibrium. To avoid this difficulty, the author provided a convexified version of his model, by replacing the consumption set by its convex hull and using the demand correspondance as in [38].

Some economists provided models *à la* Tiebout using a discreet number of household instead of a continuum. For instance, Wooders ([60] and [61]) generalized Westhoff's model, by relaxing some assumptions such as the non-rivalty of the local public goods. She assumed that the public good suffers from congestion (called here "crowding effect"), which means that households' utility decreases with the number of people that consume the public good, for any given amount of public good produced. Those papers are relatively closer to Tiebout's intuitions in the sense that crowding effet are allowed in Tiebout's paper to justify his assumption that jurisdictions would not be too crowded.

Another way to model Tiebout's intuitions is to use hedonic coalitions. Contrary to classical cooperative game theory, in which households' utility only depends on the payment they receive, in hedonic coalition framework, households also value the coalitions themselves (see for instance [3]). Consequently, one household may prefer a coalition in which he would receives a lower payment than in another one, if it prefers this coalition for its intrisec characteristics to

the other one. Dreze & Greenberg [17] proposed a model in which households are assigned to a coalition, but are allowed to quit their coalition to another one. They consider in turns the case where no transfer are allowed between coalitions, and the case where those transfers are allowed.

The models explored in the previous subsection do not integrate the land market, so households do not consume housing, and do not pay any cost for living in a crowded jurisdiction. As a consequence, since the public good does not suffer from congestion, one may assume that, in many cases, households would be better-off if they merged into an unique jurisdiction to produce a large amount of public good at a lower cost (since the aggregated wealth would be higher) instead of staying separated into different smaller jurisdictions, as long as they do not differ "too much" on the tax rate that must be applied.

This assumption is quite unrealistic: one can observe the co-existence of big cities and small villages inside an urban area. This co-existence can be explained by the presence of a competitive land market: living in a big cities allows households to consume a large amount of public good, but housing is costly. On the contrary, the housing price is lower in a small jurisdiction, but the jurisdiction will provide a lower level of public good.

The introduction of a land market in local public goods economy models will be discussed in the next subsection.

2.2 The land market

Integrating a competitive land market into a model *'a la* Westhoff raises the question of the taxation scheme. In most countries, national taxation is based on households' wealth, while local taxation is based on the housing value. Those two taxation schemes may have different implications in terms of the existence of an equilibrium within the same meaning as Westhoff, in terms of social welfare, in terms of segregative properties...

Rose-Ackerman was among the first economists to introduce a competitive land market into a local public goods economy model [51]. Her article integrated a land market in a model *a la* Westhoff. Households differ by their wealth and by their preferences. Their utility functions depend on the amount of public good in their jurisdiction Z, on the amount of the composite private good x they enjoy, and also on the amount of land h they consume, whose before tax unit price, different from one jurisdiction to another one, is denoted p_j. As previously, households vote to determine their jurisdiction's tax rate, the difference is that the tax is linearly based on the housing value. The price paid by a household for one unit of housing in jurisdiction j is then $(1 + d_j)p_j$, where d_j is the dwelling tax rate applied by jurisiction j. Hence, the amount of public spending is $d_j p_j H_j$, where H_j is the total amount of land in jurisdiction j, that

belongs to absentee landlords. In all jurisdictions, land is homogenous and perfectly divisible.

To incorporate the land market and the dwelling tax to the definition of the stability, we define $B_{ij} \in \mathbb{R}^2_+, B_{ij} = \{(x,h) \in \mathbb{R}^2_+ : x + (1+d_j)p_j \leq \omega_i$ as the set consumption of private good and housing that respects the budget constraint faced by a household i living in jurisdiction j. The introduction of a land market requires to provide a definition of a voting equilibrium, whose existence is a necessary condition to have a jurisdictions structure stable.

Definition 1 *A voting equilibrium is a tax rate, a before tax housing price and an amount of public good such that:*

- *Consumption of private good and housing is optimal for every household:* $\forall x,h \in B_{ij}, U(Z_j, \bar{x}, \bar{h}) \geq U(Z_j, x, h)$*, where* $(\bar{x}, \bar{h}) \in B_{ij}$ *is the current consumption,*
- *Every jurisdiction's public good is fully financed by the dwelling tax revenues:* $\forall j \in J, Z_j = d_j p_j H_j$*,*
- *The voting process must have a solution in every jurisdiction: for all* $j \in J$*,* t_j *must be such that 50% of the households living in* j *prefer* t_j *to any higher tax rate and 50% that prefer* t_j *to any lower tax rate,,*
- *Housing market clear:* $H_j = \int\limits_{i \in I_j} h_i d\lambda$*,* h_i *being the amount of housing consumed by household* i

With a land market, jurisdiction structure is stable if and only if:

1. A voting equilibrium must arise.
2. No household should have incentive to leave unilaterally its jurisdiction: $\forall j, j' \in J, \forall i \in I_j, \forall (x,h) \in B_{ij}, U_i(Z_j, \bar{x}, \bar{h}) \geq U_i(Z_{j'}, x, h)$.

This notion of stability incorporates the one defined in the previous subsection, but adds two elements: the land price must equal supply and demand in every jurisdiction, and be such that households would have no incentive to modify their current consumption of private good and housing. Those extra conditions make the definition of stability stronger than the one without land market.

As in Westhoff's article, local public goods do not generate spillovers in other jurisdictions and do not suffer from congestion effect. However, contrary to Westhoff, local public goods are assumed to be produced through a technology that may not be linear. The cost for producing an amount Z of public good is given by $C : \left| \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ Z & \longmapsto & C(Z) \end{array} \right.$ which is continuous and increasing.

Hence, there exists a function $C^{-1} : \left| \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ dpH & \longmapsto & C(dpH) = Z \end{array} \right.$ that gives the amount of public good that will be produced with a budget of dpH.

In every jurisdiction, the tax rate is chosen by majority voting. Households determine their favorite tax rate considering that p_j is given and that households have perfect information over the public good cost function. The new budget constraint is given by $x + (1+d)ph \leq \omega_i$. As previously, there si no savings and the utility function is assumed to be always increasing with respect to x and h, so the budget constraint will always be saturated, hence one has $x = \omega_i - (1+d)ph$. Household i's utility function can then be re-written as follows:

$$U_i(Z, x, h) = U_i(C^{-1}(dpH), \omega_i - (1+d)ph, h)$$

Contrary to articles in which taxation was based on households' wealth, a voting equilibrium may not arise: suppose that households' favorite tax rate increases as the before tax housing price decreases, that the supply is perfectly inelastic (so the supply is constant with respect to the price, which would be the case if there are a sufficiently high number of suppliers and no cost for renting land), and that no interjurisdictional migration are allowed. As a consequence, after an increase of the tax rate, the suppliers will decrease the before tax price to keep the demand constant. Then households will vote for a higher tax rate, and so on and so far, so the tax rate will tend to $+\infty$ and the before tax price, to 0, so no voting equilibrium will arise.

Let denote the indirect utility conditional upon the amount of public expenditure $E = dpH$, which is a function of the amount of public good, the net-of-tax $P = (1+d)p$ and the private wealth, as

$$V_i(E, P, \omega_i) = \max_{h \in [0; \frac{\omega_i}{P}]} U_i(C^{-1}(E), \omega_i - Ph, h)$$

The MRS of the public good by the net-of-tax housing price is given by

$$a_i = \frac{\frac{\partial V_i(E,P,\omega_i)}{\partial E}}{\frac{\partial V_i(E,P,\omega_i)}{\partial P}}$$

Households are ranked according to a_i from the lowest to the highest: $i < i' \Rightarrow a_i < a_{i'}$. Several types of equilibria may exist. For example, there can exist a stable structure in which households are divided between the jurisdictions such that, in all jurisdictions, median voters have the same MRS of the public good by the net-of-tax housing price, so that the amount of public good and the net-of-tax housing price are identical in all jurisdictions.

Another possible stable structure is the one in which households are grouped according to their preferences for the public good. Thus, each jurisdiction would be only composed of households with the same a_i. Such an equilibrium can exist

only if the number of jurisdictions is at least equal to the number of household types.

Finally, there can be mixed equilibria in which all jurisdictions have identical amounts of public good and net housing price, certains being composed of several types of households, but such that their median vote is of the same type, and others composed of only one type of household.

Equilibria in which several jurisdictions have the quantity of public good and the same unit gross price of the of land are not particularly interesting, because, in such cases, the fusion of these jurisdictions would not changed its households' utility, since the per capita amount of public good would be the same. So let only explore equilibria where two jurisictions have different amounts of public good and net housing prices.

Suppose that every jurisdiction is composed of a connected subset of the continuum of households: jurisdiction 1 is composed of the interval $[0; j_1]$, jurisdiction 2, of the intervalle $[j_1; j_2]$,..., and jurisdiction M, of the intervalle $[j_{M-1}; 1]$. Obviously, at the equilibrium, one can not have 2 jurisdictions j and j', with $j < j'$ such that $Z_j > Z_{j'}$ and $P_j < P_{j'}$, because every households would prefer to live in j. Symmetrically, one can not have $Z_j < Z_{j'}$ and $P_j > P_{j'}$. Futhermore, jurisdiction j''s median voter would not choose a tax rate that would lead to an amount of public good less than Z_j, because households living in j', by definition, have their MRS greater than households living in j, so clearly $Z_j < Z_{j'}$ and $P_j < P_{j'}$.

The proof of the existence of an equilibrium is highly challenged when a land market is introduced. For instance, Rose-Ackerman failed to identify sufficient conditions to ensure the existence of an equilibrium using Kakutani's fixed point theorem, because of the non-convexity of the consumption set. Let us consider a household endowed with a private wealth of 100 and living in a jurisdiction with $H = 100$ and $p = 2$. The cost function is given by $C(Z) = \sqrt{Z}$. The consumption bundles B_1, composed of amounts of public of 10000, of private good 40 and of housing 20, and 2 composed of amounts of public of 40000, of private good 60 and of housing 10, both respect the budget constraint $x + (1 + d)p = 100$, the first one, by applying a tax rate of $\frac{1}{2}$, so the net-of-tax housing price is 3, which respect the budget constraint, the second one, by applying a tax rate equal to 1, so the housing price is 4. However, a linear combination of budget 1 and 2, given by $Z = 25000$, $x = 50$ and $h = 15$ is not feasible, which proves that, in this case, the consumption set is not convex.

Another possible modeling of land is the one proposed by Dunz [18]: housing is indivisible, but there exist several type of housing within jurisdiction. Using this modeling, sufficient conditions to ensure the existence of an equilibrium can be found.

Another question raised by the introduction of land is its hedonic value, as developped by Rosen [52] that may depends on its location: people may be willing to spend more money for a same amount of housing in a jurisdiction 1 than in a jurisdiction 2, even if both jurisdictions offered the same amount of public good with the same tax rate, because of their intrinsec value, for instance if jurisdiction 1 is nearby the sea, while jurisdiction 2 is not.

Some authors tried to take into account the jurisdiction's intrinsec value into their analysis, as Greenberg [30] and Nechyba [46]. Nechyba proposed a model with Dunz's land market modeling ([18] and [19]) and taxation on land in which households' utility depends on their place of residence for itself, besides the public good policy applied, and in which public good in one jurisdiction can generate spillovers in others. However, among the sufficient conditions he identified to ensure the existence of a equilibrium, and also segregation, one is the indifference among 2 jurisdictions offering the same among of public good and in which the net-of-tax housing price is the same. Since this condition is sufficient to ensure the existence of an equilibrium, but has not been proved to be necessary, assuming that jurisdictions may have intrinsec value does not improve the results.

Other articles, by Epple, Filimon and Romer [22] and [23], find necessary and sufficient conditions to ensure the existence of an equilibrium, if restrictive assumption are made on the preferences and on the public good production function: the MRS of the public good by the net-of-tax housing price must be alwys increasing with the private wealth, the public good cost function must be an strictly affine function (and not a linear function), and the Marshallian demand for housing must not depend on the available amount of public good.

If taxation, as in Westhoff's article, is still based on wealth, then Konishi [43] proved the existence of an equilibrium in a model with housing market, using Kakutani's fixed point theorem. In his model, as in Rose-Ackerman's, housing is perfectly divisible.

The different possible modeling of the housing market in multijurisdictional economy models has unquestionably an impact on the existence of an equilibrium. However, it seems that the equilibrium, if it exists, will be stratified, either in terms of preferences for the public good, or in terms of wealth.

Now that the existence of an equilibrium can be ensured, let explore the question of the its propertis in terms of Pareto-efficiency.

2.3 Are equilibria always Pareto-efficient?

In this section, we will examine the properties of the equilibrium in models à la Tiebout in terms of Pareto-efficiency. Let us first remind the mostly used

notion for stability.

Definition 2 *A jurisdictions structure is stable if and only if:*

- *No household i has incentive to modify its current consumption bundle X_i or to leave unilaterally its jurisdiction for another one: $\forall j, j' \in J, \forall i \in I_j, U_i(Z_j, X_i) \geq V_i(Z_{j'}, P_{j'}, R_{ij'})$ where $V_i(Z_{j'}, P_{j'}, R_{ij'})$ represents the maximal utility that household i can obtain when the amount of public good is $Z_{j'}$, the net-of-tax prices of every private goods $P_{j'}$, and the available wealth is $R_{ij'}$.*
- *A voting equilibrium is reached in every jurisdiction.*
- *Every jurisdiction presents a balanced budget: the local public goods are fully financed by the fiscal revenues, that are devoted only to the production of the local public goods.*

The definition of Pareto-efficiency (or Pareto-optimality) is widely known: an allocation of public and private goods is Pareto-efficient if no other conceivable allocation of public and private goods can increase strictly the utility of one household, while all the others are not worse-off. A jurisdictions structure is Pareto-efficiency if it leads to a Pareto-efficient allocation of public and prvate goods.

Bewley [6] presented several counter-examples in which either no equilibrium exists, or, if an equilibrium exists, it is not Pareto-optimal. He distinguished pure public goods, that do not suffer from congestion and pure public services, which cost is proportional to the number of users. He also consider in turns the possible objective followed by local governments: a democratic one, whose goals is to maximize a certain social welfare function, and an entrepreunarial one, which is independent from households' welfare (for example, maximizing the probability of being re-elected, or the number of inhabitants).

In order to prove the existence of an equilibrium, most authors used either Brouwer's or Kakutani's fixed point theorem. Some conditions are required to allow the use of a fixed point theorem. For instance, Westhoff's main result is the identification of a sufficient condition to have all stable jurisdictions structure segregated, in addition to the standard properties assumed on the preferences: the Marginal Rate of Substitution (MRS), given by $\frac{\frac{\partial U_i}{\partial Z}}{\omega_i \frac{\partial U_i}{\partial x}}$, must be a continuous and increasing function with respect to i.

If this condition is satisfied, then an equilibrium will be reached. But the equilibrium is not necessarily Pareto-optimal. Consider any case where there is only one type of households, with half living in one jurisdiction, and the other half, in a second one. Clearly, those jurisdictions will be identical, since the tax

bases will be the same, so will be the median voters' preferred tax rate (actually, every household will be the median voter). Consequently, no household can be better-off by moving from its jurisdiction to the another one, so such a jurisdiction structure is stable. However, the jurisdictions structure where all households are merged into a grand jurisdiction would make all households better-off, since the tax base would be multiplied by 2 and every household would be the median voter, so the tax rate implemented would be the tax rate that maximizes its utility.

Futhermore, Hamilton [35] claimed that the conditions under which the provision of local public goods is Pareto-optimal according to Tiebout, such as the number of households in every jurisdiction that is assumed to be "optimal", which means that the desired amount of public good can be produced at the lowest average cost, do not hold in reality. His main contribution is to show that a land-restriction policy must be implemented to ensure the existence of a Pareto-optimal equilibrium: to be a member of the jurisdiction, a household must consume more than a defined threshold of housing.

The existence of spillovers and congestion, along with the differences in preferences, can be part of the reason why it may be efficient to have households separated between different jurisdiction, instead of having all households merged into a grand jurisdiction. Let us first present a definition of spillovers.

Definition 3 *A local public good produced by jurisdiction j generates positive spillovers in another jurisdiction j' if and only if the available amount of public good in j' is strictly increasing with respect to the amount of public good produced in j.*

Among the several possible way to model the spillovers, Greenberg's modeling [30] is the more general: households' utility function depends on the amount of public good produced by the jurisdiction and on the number of households that live in it. The impact of the mass of the population on the public good is not defined, which is a very general, and an interesting way to model the congestion effects. Greenberg [30] also generalized Westhoff's model by introducing the presence of profit-maximizing firms that produce several kinds of private goods, by relaxing the assumption that local taxes are proportional. The only assumption made on the tax scheme is that no households would be asked to paid a tax greater than its wealth. However, under standard assumptions, Greenberg proved that the existence an equilibrium can be ensured.

Papers such as [9] and [8] provide a modelling for spillovers generated by local public goods: the available amount of public good households living in jurisdiction j can enjoy is given by

$$Z_j = \sum_{k \in J} \alpha_{jk} \zeta_k$$

where α_{jk} is the spillovers coefficient from jurisdiction k to jurisdiction j. The spillovers coefficient matrix $A = [\alpha_{jk}]$ does not need to be symmetric, the only assumption is that, $\forall (j,k) \in J^2, \alpha_{jj} = 1$ and $0 \leq \alpha_{jk} \leq 1$.

Using a discrete number of households could have caused problems to prove the existence of an equilibrium. The problem is known as "the integer problem" [60]. Since Brouwer's theorem can be applied only to continuous function, it mazy not be applied if the set of households is not continuous, as if it the case here. However, Wooders, by replicating the economy, found a solution to prove the existence of an equilibrium.

The introduction of congestion and spillovers gives the intuition why having households separated between different jurisdiction may be unanimously preferred to the grand jurisdiction, while models *a la* Westhoff conclude that the grand jurisdiction is always a Pareto-efficient equilibrium.

Definition 4 *Local public goods suffer from congestion effects if and only if the available amount of local public goods households can enjoy, Z, depends positively on the public expenditure E and negatively on the mass of households that can consume them, μ_j.*

There exist several approaches to model congestion. One possible but extreme way of modeling congestion is to assume that the public good cost function depends on the per capita amount of public good. It is equivalent to assume that households share the local public good equally, so, if the taxation is based on wealth, the available amount of public good each household can consume is given by $Z_j = \frac{t_j \varpi_j}{\mu_j}$ where $\mu_j = \int -i \in I_j d\lambda$ is the mass of households in jurisdiction j. The local public good can then be considered as a perfectly rival common ressource. Some articles modeled congestion in such a way (see for instance [11]).

Such a modeling is quite restrictive and unrealistic: a million households endowed with the same wealth would enjoy the same amount of public good if all lived in a grand jurisdiction as if they all loved separately into a million jurisdictions. This modeling can be applied only to a very few kind of public good, for exemple a road.

A more realistic modeling is the one provided by Oates [47]. Oates modeled the available amount of public good $Z_j = \frac{\zeta_j}{\mu_j^\beta}$, where ζ_j is the amount of public good produced by jurisdiction j, and empirically estimated the parameter β. Though, for most public goods, the parameter β will be included between 0 and 1, there exist some kinds of public good for which the hypothesis $\beta = 1$ can not be excluded.

If local public goods do no suffer from congestion effects, the grand jurisdiction is always Pareto-optimal, for a simple reason: the median voter(s) in the grand jurisdiction will always be worse-off if there are at least 2 disctinct jurisdictions, because the tax base of its jurisdiction will be lower, so even if it is still the median voter, its utility will be lower. The same reasoning can be used to demonstrate that an equilibrium in Wooders' model may not be Pareto-optimal if the crowding effect is not too strong.

However, if congestion effects are strong enough, then the grand jurisdiction might not be Pareto-efficient: Suppose that the local public good is perfectly rival, so the available amount of public good is the amount of public expenditure divided by the mass of households, and that there exist two groups of households, having the same wealth, but different preferences, one group wants a high tax rate and a high amount of public good, while the other group would choose a lower tax rate. The jurisdiction structure composed of the grand jurisdiction is clearly not Pareto-efficient, because the group having the greatest mass will choose the tax rate. If we consider the jurisdictions structure where the two groups were separated into two disctinct jurisdictions, the tax base would be the same in every jurisdiction, and both groups could have its favorite tax rate. Consequently, households belonging to the group with the greatest mass would be indifferent between living in the grand jurisdiction and living in a jurisdiction composed only of other households of its group, while households belonging to the other group would strictly prefer the second jurisdictions structure to the first one.

However, the grand jurisdiction is not necessarily socially efficient, as shown by Guesnerie & Oddou ([33] and [34]). They used the notion of superadditivity developped in the cooperative game theory. A game is superadditive if and only if the payoff of the union of two coalitions is greater than the sum of the two separated coalitions' payoffs. Applied to the jurisdiction formation theory, the superadditivity means that two jurisdictions could produce a greater amount of public if they merged than the sum of the amount of public good they could produce in they remain separated apart. They proved that, in a model *à la* Westhoff, if the game is superadditive, then the grand jurisdiction is always efficient according to any individualistic social welfare function. If the game is not necessarily superadditive (which would be the case is there exist some congestion effects), clearly it might not be the case.

Their model is quite unperfect though, because the existence of an equilibrium can not be proved. Konishi, Le Breton & Weber [44] even show that such a model may not admit a pure strategy Nash equilibrium if the local tax based on wealth is proportional and preferences are quasi-linear with respect to the private consumption. A pure strategy Nash equilibrium is a jurisdictions structure where no household have incentive to move unilaterally to another jurisdiction nor to create its own jurisdction.

If there exist several local public goods, then there may not exist a voting equilibrium within a jurisdiction, so, even if mobility is not allowed, or if there exists only one jurisdiction, the stability of a jurisdictions structure could be compromised. To ensure the existence of a voting equilibrium if there exist q local public goods, a voting rule has been identified by Greenberg [29] and used later by Greenberg & Shitovitz [31]: the q-majority rule.

This rule states that an existing allocation $(Z_1, ..., Z_q)$ of local public goods will be replaced by another feasible allocation $(Z'_1, ..., Z'_q)$ if and only if the fraction of households that prefer the second allocation to the existing one is greater than $\frac{q}{q+1}$. Obviously, if $q = 1$, the q-majority rule degenerates to the simple majority rule.

Most articles considering the existence of several public goods use this rule in order to ensure the existence of a voting equilibrium, such as Konishi [43]. Konishi's proof of the existence of an equilibrium has been adapted in other articles to prove the existence of an equilibrium, for example by Bloch & Zenginobuz in [9].

The authors, using Konishi's proof of the existence of an equilibrium, ensured the existence of an equilibrium under 2 assumptions: the public good and the composite private good are both normal (which is a pretty standard assumption), and the MRS of the private good for the public good tends to 0 when the available amount of public good tends to 0, and, for an available amount of public good $\bar{Z} \in [0; 1]$, tends to something greater or equal to $\bar{Z}$.

Moreover, if the spillovers coefficient matrix is symmetric, the authors proved that all stable jurisdictions structure are symmetric, while if it is asymmetric, there exist an unique equilibrium if spillovers coefficients are low enough, and several equilibria if they are high enough.

3 The definition of segregation

Among economists, sociologists, political scientists, and more generally social scientists, the discussion on segregation by income between neighborhoods or communities became more and more important as more and more people moved to a metropolitan area. Debates exists about whether or not segregation is ineluctable, efficient or even desirable from a welfarist point of view.

Although there exist lots of arguments against or in favor of segregation by wealth (or by income), one may just argue that households who are similar in terms of wealth would have the same preferences over the public good, and so decisions would be easier to take and more respected. On the contrary, one may object that poverty concentration in a neighborhood seems to be correlated

with high criminal activity, not only in this very neighborhood, but in the whole metropolitan area.

More generally , the existence of an impact of spatial inequalities on metropolitan areas' growth and development is widely admitted by economists and social scientists. However, this section will not provide arguments for or against segregation, but present the possible definitions of segregation and its causes. One possible way to define segregation is to measure it through an index.

Although there exists several indexes measuring segregation within groups if there are only two different types (blacks and whites, or males and females), such as Duncan's (1955), Theil and Finizza's (1971), Hutchens' (2000) or Frankel and Volij's (2005), only few indexes that can measures segregation between more than two types, or with a continuous variable. Let's just mention the Neighborhood Gini Coefficient [39], the Neighborhood Sorting Index (NSI) [40], the Gini Coefficient of Segregation (NCS) [41] and the Centile Gap Index (CGI) [58].

We denote μ as the mass of households in the area, and μ_j as the mass of households living in jurisdiction j, ω^{mean} (resp. ω^{med}) as the mean (resp. median) income in the whole area, and ω_j^{mean} (resp. ω_j^{med})as the mean income (resp. median) in jurisdiction j. Finally, we denote σ_j as the standard income deviation in jurisdiction j, and σ, the standard income deviation in the whole area. A jurisdictions structure is defined as the triplet $S = (J, \{I_j\}_{j\in J}, \{\omega_i\}_{i\in I})$

The segregation index of a jurisdictions structure S is a function $I(S) \in [0; 1]$, the closer $I(S)$ is to 1, the more the jurisdictions structure S is segregated. Of course, to compute a segregation index, we will require that the considered area is composed of at least 2 jurisdictions and at least 2 different levels of wealth.

Let's formally write the previously mentionned income segregation indexes :

- The Neighborhood Gini's Coefficient (NGC) : $NGC = \frac{M+1}{M-1} - \frac{2}{M(M-1)\omega^{med}} \sum_{j=1}^{M} R_j \omega_j^{med}$, where R_j is the rank of jurisdiction j, such that the rank of jurisdiction with the highest median wealth is 1, and the rank of jurisdiction with the lowest median wealth is M.

- The Neighborhood Sorting Index (NSI) : $NSI = \frac{\sum_{j=1}^{M} \mu_j (\omega_j^{mean} - \omega^{mean})^2}{\sigma^2}$.

  The NSI is the ratio Variance inter-jurisdiction over total variance.

- The Gini Coefficient of Segregation (GCS) : $NCS = \frac{\int_{i=0}^{1} (\int_{i'=0}^{1} |\omega_{ni} - \omega_{ni'}| di') di}{\sum_{i=0}^{1} (\sum_{i'=0}^{1} |\omega_i - \omega_{i'}| di') di}$
  where ω_{ni} is the mean wealth in i's jurisdiction.

- The Centile Gap Index (CGI) : $CGI = 1 - \frac{4\int_{i=0}^{1}|P_i - P_i^{med}|}{\mu}$ where P_i is the estimated percentile in the whole area distribution of household i , and P_i^{med} is the estimated percentile in the whole area distribution of the median household of the jurisdiction in which household i lives.

So far, no segregation index respecting several desirable properties has been identified. Among the desirable properties a segregation index should respect, there are:

- Scale Invariance: $\forall k \in \mathbb{N}^*, \forall S, S^k$, where $S = (J, \{I_j\}_{j\in J}, \{\omega_i\}_{i\in I})$ and $M^k = (J^k, \{I_j^k\}_{j\in J}, \{\omega_{i^k}\}_{i^k\in N^k})$ such that $i \in I_j \Rightarrow \forall i^k \in [k(i-1)+1; ik], i^k \in N_j^k$ and $\forall i, \forall i^k \in [k(i-1)+1; ik], \omega_{i^k} = \omega_i$ then $I(S) = I(S^k)$. In words, a segregation index that respects this property will remains constant if each household is "cloned" k times. This axiom is necessary to compare 2 metropolitan areas of different population sizes.

- No Monetary Illusion: $\forall c \in \mathbb{R}_*^+, \forall M, M'$ where $M = (N, J, (N_j)_{j\in J}, (\omega_i)_{i\in N})$ and $M' = (N, J, (N_j)_{j\in J}, (c\omega_i)_{i\in N})$, then $SI(M) = SI(M')$. This axiom is useful to compare 2 metropolitan area from different monetary zones or to observe the evolution of the segregation within a metropolitan area between two periods, to avoid the inflation issue.

- Sensitivity to per capital wealth differences: $I(S) = 0 \Rightarrow \omega_j^{mean} = \omega_{j'}^{mean}, \forall (j, j') \in J^2$. Consider a metropolitan area M composed of 2 jurisdictions. The first jurisdiction is composed of 25 homeless households, with no wealth, 1 household endowed with 10 K\$ and 25 households endowed with 15 K\$, while the second jurisdiction is composed of 25 households endowed with 5 K\$, 1 household endowed with 10 K\$ and 25 households endowed with 20 K\$. Here, $\varpi_1 \approx 7.55$\$ and $\varpi_2 \approx 12.45$\$, but both the NGC and the CGI will be equal to 0.

- Sensitivity to skewness differences: $I(S) = 0 \Rightarrow \gamma_j\gamma_{j'} > 0, \forall (j, j') \in J^2$, with $\gamma_j = \frac{\omega_j^{mean} - \omega_j^{med}}{\sigma_j}$ being Pearson's second skewness coefficient for jurisdiction j. This property is desirable for many reasons, the most important one being the difference in public good provision between 2 jurisdictions having the same average wealth, but differents distribution skewness. This property is not respected by the NGC and the CGI. In the last example, $\gamma_1 \approx -0.33$ and $\gamma_2 = \approx 0.33$, so $\gamma_1 \neq \gamma_2$ but NGC=CGI=0. The NSI and the GCS don't respect this property neither. Let's consider a metropolitan area M composed of 2 jurisdictions. The first jurisdiction is composed of 20 homeless households, with no wealth, and 80 households endowed with 10 K\$, while the second jurisdiction is composed of 99 households endowed with 5 K\$ and 1 household endowed with 305 K\$. In this example, $\gamma_1 = -\frac{1}{2}$ and $\gamma_2 = \frac{1}{3\sqrt{11}}$, so $\gamma_1 \neq \gamma_2$. Those 2 jurisdictions

have the same average and aggregated wealth, so NSI=GCS=0, but they would certainly not have the same type of public good, schools will not be frequented by the same population of students...

- Monotonicity: $\forall S^A = (J, (I_j^A)_{j\in J}, (\omega_i)_{i\in I}), S^B = (J, (I_j^B)_{j\in J}, (\omega_i)_{i\in I})$, where :
  1. $i \in I_j^A, i' \in I_{j'}^A, \omega_i > \omega_{i'}$,
  2. $\omega_{j^A}^{mean} < \omega_{j'^A}^{mean}$,
  3. $\omega_{j^A}^{med} \leq \omega^{med}$,
  4. $\omega_{j'^A}^{med} \geq \omega^{med}$,
  5. $I_j^B = I_j^A \cup \{i'\} \setminus \{i\}, I_{j'}^B = I_{j'}^A \cup \{i\} \setminus \{i'\}$,
  6. $\omega_{j^B}^{med} \leq \omega^{med}$,
  7. $\omega_{j'^B}^{med} \geq \omega^{med}$,

  $I(S^A) < I(S^B)$.

This last property implies that any population movement, which increases the difference in average wealth between jurisdictions, without changing the relative position of their respective with respect to the medain wealth of the whole area, should increase the segregation index too. The NGC violates this properties. Let's consider a metropolitan area M composed of 2 jurisdictions. The first jurisdiction is composed of 20 homeless households, with no wealth, 10 households endowed with 10 K$, and 20 households endowed with 11 K$ while the second jurisdiction is composed of 20 households endowed with 5 K$, 10 household endowed with 11 K$ and 20 households endowed with 15 K$. Replacing one household from the first jurisdiction endowed with 11 K$ with one household from the second jurisdiction endowed with 5 K$, neither the median nor the rank of each jurisdiction will change, so neither would the NGC.

The CGI does not respect the third property, while the NSI and the GSC do not respect the fourth one and the NGC, the third and the last one. Moreover, to be able to compute the GSC and the CGI, one must know the exact distribution of wealth in every jurisdiction, which may be very costly.

For those reasons, from the best of my knowledge, economists who studied the causes of segregation do not use either one of those measures in theoretical models.

Another possible definition of segregation can be expressed in terms of consecutiveness of the continuum of households, as it is known in coalition theory, see for example [32]: a jurisdictions structure is segregated if and only if every

jurisdiction is composed of a connected subset of the continuum of households.

In plain English, this definition means that a jurisdictions structure is segregated by wealth if and only if, for every pair of jurisdictions, the richest households of the jurisdiction with the lowest per capita wealth is poorer than the poorest households living in the other jurisdiction. Clearly, this is an extrem notion of segregation, but that also makes it interesting, because it allows economists to identify the the segregatives forces that would lead to such a pure segregation in a stylized world.

In most articles dealing with the wealth-stratification of stable jurisdictions structure, authors consider this definition of the the segregation. Although it presents some default, one of them being the fact the grand jurisdiction, i.e. a jurisdiction structure composed of only one jurisdiction gathering all households, would be considered as segregated.

Despite our awareness of this definition's imperfections, from now, that is the one we will use in the rest of the survey.

4 The segregative properties of endogenous jurisdictions structure formation in the theoretical literature

The first article that provided a condition to ensure the wealth-stratification, expressed in terms of the consecutiveness of the set of households when ordered by their wealth, of any stable jurisdictions structure is from Ellickson[20]. This condition is the single crossing of households indifference curves in the "tax-rate-amount of public good" space. One can notice that, as we previously mention it, this condition is also sufficient to ensure the existence of an Equilibrium.

This definition is widely used in the theoretical literature on segregation. However, this extreme notion of segregation does not allow to compare two different areas, or to examine the evolution of an urban area at two different periods, unless one has all its jurisdiction compose of connex subsets and not the other. Moreover, let's consider an urban area composed of millions of inhabitants that is segregated in a certain period according to Ellickson's definition. Suppose that in next period, two households from different towns exchange their respective housing. Then the area is not considered as segregated anymore, although the change is relatively minor.

One strong assumption of models *à la* Tiebout is the fact that households choose their place of residence according to a trade-off between the amounts of public good provided by the jurisdictions and the tax they would pay there.

According to Percy and ali[49], this assumption can not be rejected. By using data on interjurisdictional migrations among 55 communities, they show that the decision of voting with one's feet and the choice of the new jurisdiction depend greatly on the levels of tax rates and of the amount of public services available in the different jurisdictions.

Empirical evidences brought by Banzhaf and Walsh[4] also show that households actually choose their place of residence according to local amenities that could be improved by the presence of a local public good. In particular, rich households are willingto pay higher taxes in order to enjoy high air quality.

Another article [28] identified a necessary and sufficient condition on the preferences to have all possible stable jurisdictions structures segregated in a model very close to Westhoff's: there is a continuum of households who differ by their wealth, they vote over the tax rate applied by their jurisdiction, and they can leave their jurisdiction for another that would provide a higher utility level. However, contrary to Westhoff's model, no specific collective decision rule is imposed, the authors only assume that the tax rate applied by th jurisdiction is democratically chosen, that is to say it is between the lowest tax rate preferred by one households and the highest tax rate preferred by another one.

The main difference between the articles is that households have identical preferences over the public good and the composite private good. This assumption, while restrictive, enables the authors to identify a necessary and sufficient condition to ensure the segregation of every stable jurisdictions structure: the Gross Substitutability/Complementarity (GSC) condition.

The public good is a gross substitute (resp. a gross complement) if and only if, $\forall p_Z, p_x, \omega_i \in \mathbb{R}_+, Z^M(p_Z, p_x, \omega_i)p_x > 0$ (resp. < 0). The GSC condition is necessary in the sense that, for any violation of the condition, one can always construct a stable and yet no segregated jurisdictions structure. However, it does not mean that a stable jurisdictions structure could not be segregated if the GSC condition was violated. It is also sufficient because it is equivalent to the sufficient condition identified by Westhoff.

While stringent, this condition is not outlandlish. It is equivalent to have the preferred tax rate being a monotonic function of the private wealth, for any given amount of aggregated wealth. The validity of this condition can be challenged by several generalization of the model. For instance, so far, households are supposed to be freely mobile. Obviously, if the cost for moving from one jurisdiction to another is extremely high, no household would leave its jurisdiction, even if another jurisdiction implemented a more desirable package "tax rate-amount of public good".

When households share the same preferences, the GSC condition implies the condition identified by Westhoff to ensure the existence of an equilibrium.

More precisely, under the assumption that households have the same utility function, the condition identified by Westhoff is equivalent to have the public good to be a gross complement to the composite private good (see [28] lemma 5).

Consequently, Gravel and Thoron's contribution is to identify a condition that is not only sufficient, but also necessary to have every stable jurisdictions structure segregated by wealth, while Westhoff and other authors only identify sufficient conditions. However, Westhoff's model is more general, since households may have different preferences.

5 Do people self-sort themselves into homogenous jurisdictions? : Empirical results

5.1 The segregative factors

A reasonable definition of segregation is still to be invented, in order to provide empirical tests of the existing theoretical models. However, there exist some attempts to prove empirically that jurisdictions structures are segregated, such as Bergstrom and Goodman's article [5].

In this article, empirical evidences suggested that the richer a household is, the more it is willing to pay taxes so as to increase its consumption of public good, for any other given parameters. It means that a household's preferred tax rate is a monotonic function of its private wealth, which is equivalent to the condition identified by Gravel & Thoron to ensure the wealth stratification of any stable jurisdictions structure.

Those results are confirmed by another empirical article by Deacon and Schapiro[15] dealing with households' votes. The authors demonstrated that households' predisposition to vote in favor of the improvement of a collective good is positively correlated with the income.

Those articles estimated the demand for the public good through aggregate data using a median voter model. The principle is quite simple: examining an eventual correlation between the provision of public good in one jurisdiction and the characteristics of the "median household" of the jurisdiction (the median income, the median political preferences...).

However, those articles suffer from the assumption that households are not mobile among jurisdictions. The imperfection is corrected by another article provided by Goldstein and Pauly[27]. In this article, they emphasize the impossibility to use a median voter econometric model to estimate the demand for the public good when households are free to leave their jurisdiction for another one, because of what they called the "Tiebout's bias": ignoring that households

may vote with their will lead to a estimated demand for the public good that will be biased in a direction that can be determined.

Even when the assumption that voters are myopic, the results remains robust, according to Epple and Sieg[25]. They tested the validity of Ellickson's condition to ensure the wealth-stratification of any stable jurisdictions structure: the single crossing of households' indifference curves in the "tax rate-amount of public good" space. They found out that this hypothesis can not be significantly rejected.

The same results exist when local taxation is progressive, according to a empirical study realized by Schmidheiny[55]. However, in his article, he found that rich households prefer to move to low-tax jurisdictions, which contradicts the previous articles.

Peer-effects may also increase the segregative properties of endogenous jurisdictions formation, if we compare the results obtained by Epple and al. in [24] and [11] showing that segregation by wealth, among households sharing the same preference, is present inn both case, both results are more significant when peer-effects are allowed.

Those results are confirmed by Alesina and ali[1]. In their article, they presented evidences that households may choose a smaller jurisdiction -that produces a lower amount of public good- in order to avoid income heterogeneity, which prove the impact of peer-effect in favor of the segregation.

Another article, by de Bartolome & Ross[12] (confirmed in [14]), provided an model based on a "monocentric city" model, *à la* Alonso[2], with 2 types of households (rich or poor) showing that, if peer effects are neither too strong nor too weak, 2 equilibria may arise, one with poor households living in the center of the city while rich people live in the suburbs, and the other where, on the contrary, rich people live in the center while poor households live in the suburbs. This model is empirically tested and consistent with the data.

To conclude this section, the main factors that favors segregation, according to the theoretical and the empirical literature, are the monotonicity of the willingness to pay for the public good with respect to the wealth, the congestion and the peer group effects.

5.2 The anti-segregation forces and the empirical evidence

In most articles, under the sufficient conditions identified to ensure the existence of an equilibrium, all stable jurisdictions structures will be segregated. However, Nechyba [46] identified extra conditions to have stable jurisdictions structure segregated, according to different possible definitions of the segrega-

tion. The extra condition to ensure the wealth-stratification of any jurisdictions structure is to have all households sharing the same preferences over the public good, the private good and the housing.

Though the model is more realistic than Gravel & Thoron's one, the condition is only sufficient, and may not be necessary, in the sense that a violation of the condition may not always allow to construct a jurisdictions structure that is both stable and non-segregated.

Consequently, not only all stable jurisdictions structure are not segregated, but even under conditions that are sufficient to ensure the existence of an equilibrium, a stable jurisdictions structure may not be segregated.

Although many models conclude that the endogenous jurisdictions structure formation leads to have any stable jurisdictions structure segregated, there does not exist a single metropolitan area that is segregated in the sense of Greenberg and Westhoff. This could mean that jurisdictions structure are not stable yet, but converge to a stratified equilibrium, so endogenous jurisdictions formation does lead households to self sort themselves into homogenous jurisdictions.

However, one mst keep in mind that Tiebout's intuitions are based on stringent assumptions, such as the absence of mobility costs, no spatial environment (and consequently no job location), the large number of jurisdictions inside a rban area, no housing market...

The relaxation of one (or several) of these assumptions may contradict Tiebout's predictions, for instance, if mobility costs were too high, no household would leave its jurisdiction, even if another one apply a better policy.

One of the first septicisms of Tiebout's predictions come from Dowding and ali[16]. They considered that Tiebout's assumptions of free mobility is not consistent with the empirical studies, and, consequently, the outcome can not be as predicted by Tiebout. Futhrmore, they questionned the validity of another Tiebout's assumption: households' choice of location could be determined not only by the tax rates and the amounts of public good produced by the jurisdictions of their metropolitan area, but also by other factors, such as the job location or the type of public good provided.

A empirical survey provided by Rhode and Strumpf[50] suggests that fiscal federalism does not lead to segregation. Even though, on one hand, their article claims that segregation increases as mobility costs decrease, on the other hand, they also asserts that their data do not present significant evidences that jurisdictions structure converge to a segregated equilibrium. This implies that there must exist factors that mitigate the segregatives properties of endogenous jurisdictions formation, such as differences in preferences, employment opportunities, or maybe other reasons. This article proved not only that endogenous

jurisdictions formation does not lead households to self-sort themselves according to their private wealth, contrary to Tiebout's intuitions, but also that it may decrease income heterogeneity accross jurisdictions.

Those results have been partially confirmed by a difference-in-difference study provided by Farnham and Sevak[26], although stronger evidences show that migrations *à la* Tiebout are present when differences between the states' taxation schemes are controlled for.

A strong force that mitigates the segregative properties of endogenous jurisdictions structure formation is the difference in terms of preferences for the public good. If, *ceteris paribus*, empirical evidences suggest that local public goods are a substitute to the private consumption, one must take into account that households can not be characterized only by their private wealth.

For instance, the willingness to pay for the public good seems to be increasing with respect the age, because elderies could become unable to drive anymore, and prefer to take the public transportation, or with respect to the number of children. Consequently, a young and poor household with no child could desire the same amount as an old and richer one, so there would be no segregation.

Others reasons that could lead to a non-stratified equilibrium are the fiscal competition, housing prices and the existence of commuting costs, as another monocentric city model provided by de Bartholome and Ross[13] suggests. In their article, 2 kinds of equilibria may arise: one segregated and one non-segregated in which rich and poor households are indifferent between living in the city with a majority of poor households (and, consequently, their preferred policy) and living in the suburbs with a majority of rich households.

Such an hypothesis is confirmed by Nechyba's article[46], in which 2 other types of segregation may arise: segregation by housing and/or by preferences, if they do not share the same preferences over the local public goods and if there exist different types of housing. Different preferences seems to mitigate the segregatives properties of jurisdictions formation *à la* Tiebout.

The proposition that differences in preferences over the housing is a factor that decreases the segregative forces is partially confirmed by Schmidheiny[56] in a two jurisdictions model: there will be an imperfect segregation, e. g. at the equilibrium, some rich households will live in the poor jurisdiction and vice-versa.

Several other articles provided a theoretical model and empirical evidences that stable jurisdictions structures are not segregated because of the differences in taste for the public good, such as [24] and [11].

In these articles, the authors developped a model in which households differ

in wealth and in preferences. Households' preferences are represented by an utility function depending on the amount of public[2] that is financed throught a dwelling tax, on the amounts of housing and of a composite private good they own, and finally on a taste parameter $\alpha \in [0;1]$ representing their preferences over the public good[3]

The differences in preferences could also decrease wealth segregation if local political institutions have other competencies than just the provision of a local public good, as claimed in Kollman and ali[42]: for instance, if a jurisdiction has the competencies to organize a referendum on alcohol prohibition, then clearly, wealth segregation may not occur, because even if the single crossing of the indifference curves in the tax rate-provision of public good holds, there can be two disctint jurisdictions both composed of rich, poor and middle-class households, one in which alcohol is legal and the other one in which it is prohibited.

To ensure the existence of an equilibrium, the authors assumed that the slopes of households' indifference curves in the "tax rate-quality of the local public good" space are increasing with respect to their private wealth and to their taste for the public good. Consequently, for households having the same preferences for the public good, two indifference curves will cross only once, so, if households have the same preferences, any stable jurisdictions structure will be segregated. However, when households do not share the same preferences, this is not the case anymore.

Futhermore, the parameter representing the taste for the public good is not significally correlated with the private wealth, which implies that there exist poor households who would vote for the same tax rate as rich people, so those households would co-exist in the same jurisdiction.

Rubinfeld and ali's article[53] provided a discussion on Tiebout bias introduced in the previous section and an attempt to correct it. Using micro data, they found out that the correlation between the demand for public schooling and the income is lower than when aggregate data are used and is not significant anymore.

We can conclude this section that the main forces that mitigate the segregative properties of endogenous jurisdictions formation are the differences in tastes, over both the public good, the housing and the location, the mobility costs and the existence of spillovers generated by local public goods in other jurisdictions.

[2] In [11], the amount of public good is replaced by the quality of the public good, that depends on the per capita jurisdiction's spendings for the public good, and also on the mean income in the jurisdiction (which measures the peer effect).

[3] for instance, if preferences are represented by a CES function, the utility function could be of the shape $U(Z,x,h) = (Z^{\alpha\rho} + xh^{\rho})^{\frac{1}{\rho}}$.

6 Conclusion

The question of the modeling of jurisdictions structure formation, inspired by Tiebout's intuitions, have been explored by several economists, in order to provide a realistic model so as to investigate on the consequence on segregation, on social welfare, and on the provision of public goods that fiscal federalism can have.

It is hard to provide a perfectly realistic model that could be applied to any country, since fiscal federalism structure is not the same from one country to another one. Those differences could be explored in order to identify what are the causes of segregation.

Another interesting question would be the definition of the segregation. Tiebout, followed by other economists, assume that households living the same jurisdictions are homogenous in terms of preferences, others define the segregation in terms of consecutiveness of the continuum of households.

Several indexes measuring segregation by wealth have been proposed, but none of them respects simultaneously several desirabl properties. Developping an index satisfying all those properties, or, alternatively, an impossibility theorem, would be a major improvement of the literature.

If most of the theoretical models based on Tiebout intuitions concluded that, under some stringent assumptions, households will self-sort themselves into homogenous jurisdictions, some of these assumptions are not consistent with empirical data, that is the reason why perfect stratification does never occur in the real life.

Finally, according to the existing literature, the main factors that favors segregation accross jurisdictions are the monotonicity of the willingness to pay for the public good with respect to the private wealth, the congestion and the peer group effects.

The factors that mitigate the segregative properties of endogenous jurisdictions formation are the differences in preferences over the public good, the presence of a competitive housing market, the mobility costs, the intrinsec value of the possible locations and the existence of spillovers generated by local public goods in other jurisdictions.

An interesting extension of this survey would consist in examining whether further generalizations of models *à la* Tiebout favor or mitigate the segregative properties of endogenous jurisdictions formation, and in testing those results empirically.

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